

Euler Characteristic Practice Problems

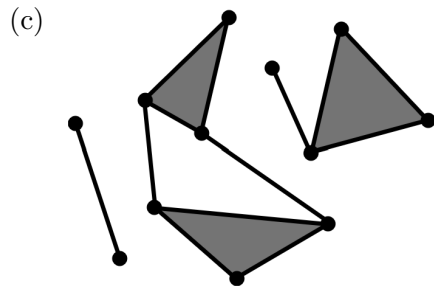
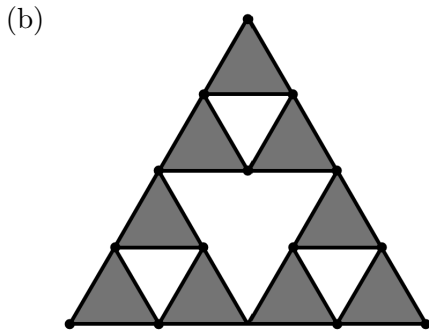
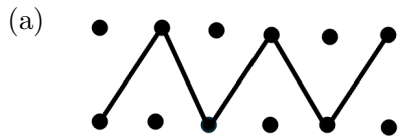
Math 384

Recall the definition of Euler characteristic: The Euler characteristic of a simplicial complex K is the integer

$$\chi(K) = \sum_{i=0}^{\dim K} (-1)^i K_i,$$

where K_i is the number of i -dimensional simplices in K .

1. Compute the Euler characteristic of each of the following simplicial complexes.



2. Give three examples of simplicial complexes with Euler characteristic 8, each with a different number of simplices.
3. Suppose that each simplex in a simplicial complex K has a unique set of vertices. That is, K may not have two different simplices with exactly the same vertex sets. Further suppose that $\chi(K) = -n$, where n is a positive integer. What is the minimum number of simplices in K ? Your answer should depend on n .
4. Prove that the Euler characteristic is additive, meaning that

$$\chi(A \cup B) = \chi(A) + \chi(B) - \chi(A \cap B)$$

for any simplicial complexes A and B .