Homework 7

Math 330

due at 5pm on Thursday, November 9, 2023

Solve the following problems and communicate your solutions clearly. Explain your work using complete sentences, and include diagrams as appropriate.

For this homework, you must type your solutions to all of these problems in LaTeX. Plots/graphs may be drawn by hand or using technology and inserted into your LaTeX document. Make sure your solutions are easy to read, in order, and clearly labeled. Upload a single file containing your solutions to the Homework 7 assignment on Moodle.

1. (3 points) Exercise 4.3.3

2. (3 points) Exercise 4.3.6

3. (8 points) Solve Laplace's equation on the square $D = \{(x, y) \mid 0 \leq x \leq \pi, 0 \leq y \leq \pi\}$ with the boundary conditions

$$u(x, 0) = 0, \qquad u(x, \pi) = \sin^3(x), \qquad u(0, y) = 0, \qquad u(\pi, y) = 0.$$

4. (4 points) Exercise 5.1.1 (a) and (b)

5. (4 points) Exercise 5.1.3 (a) and (b)

6. (8 points) The equation $\frac{\partial^2 u}{\partial t^2} + \beta \frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ represents a damped version of the wave equation. Consider this PDE with boundary conditions $u(t, 0) = u(t, 1) = 0$ on the interval $0 \leq x \leq 1$. Assume that $0 < \beta < 2c\pi$, so the damping coefficient is small relative to the wave speed.

(a) Find all separable solutions to this PDE that satisfy the boundary conditions.

(b) Find the series solution to this boundary value problem. If the initial conditions are $u(0, x) = f(x)$ and $\frac{\partial u}{\partial t}(0, x) = g(x)$, state integral formulas for the coefficients of your series solution.

(c) Discuss the long-term behavior of your solution.