1. (a) linear ODE:

linear: derivatives are not inside of other functions

$$a_n(t)\frac{d^n y}{dt^n} + a_{n-1}(t)\frac{d^{n-1}}{dt^{n-1}} + \cdots + a_1(t)\frac{dy}{dt} + a_0(t)y = F(t)$$

order; coefficient functions

(b) examples of linear ODEs:

$$\frac{dy}{dt} + 2ty = t^2$$

variable (non-const) coeff.

$$\frac{dy}{dt} = t^2 - 2t$$

const. coeff

first order

$$\frac{d^2y}{dt^2} + ty = \cos(t)$$

second order; non-homogeneous

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} = 0$$

homogeneous

2. $\frac{d^2y}{dt^2} + \frac{dy}{dt} - 6y = 0$

GUESS $y(t) = ce^{rt}$

characteristic polynomial: $r^2 + r - 6 = 0$

distinct real roots: $r_1 = -3$ and $r_2 = 2$

general solution: $y(t) = \underbrace{c_1 e^{-3t}}_{y_1(t)} + \underbrace{c_2 e^{2t}}_{y_2(t)}$