Math 262

Section 4.6 Day 35

1. Let X_1 and X_2 be uniformly distributed on the region of the x_1x_2-plane defined by $0 \le x_1$, $0 \le x_2$, and $x_1 + x_2 \le 1$. Let $Y = X_1 + X_2$. Use the following steps to find the density of Y.

   (a) Sketch the region of positive density for X_1 and X_2 in the x_1x_2-plane. Identify the possible values of Y.

   (b) Let y be a possible value of Y. Sketch the graph $Y = y$ in the x_1x_2-plane.

   (c) Find the region R in the x_1x_2-plane where $Y \le y$.

   (d) Find the cdf $F_Y(y)$ by integrating the joint density of X_1 and X_2 over the region R.

   (e) Differentiate $F_Y(y)$ to obtain the density $f_Y(y)$.

2. Let X_1 and X_2 have joint density $f(x_1, x_2) = 3x_1$, for $0 \leq x_2 \leq x_1 \leq 1$. Let $Y = X_1 - X_2$. Use the following steps to find the density of Y.

   (a) Sketch the region of positive density for X_1 and X_2. Identify the possible values of Y.

   (b) Sketch the graph $Y = y$ in the x_1x_2-plane.

   (c) Find the region R in the x_1x_2-plane where $Y \leq y$.

   (d) Find the cdf $F_Y(y)$ by integrating the joint density of X_1 and X_2 over the region R.

   (e) Differentiate $F_Y(y)$ to obtain the density $f_Y(y)$.

Here are two more practice problems you can do on other paper.

3. The joint density of X_1 and X_2 is $f(x_1, x_2) = 4e^{-2(x_1+x_2)}$ for $X_1 > 0$ and $X_2 > 0$. Find the density of $Y = \frac{X_1}{X_1+X_2}$.

4. **Challenge:** Let the point (X, Y) be randomly selected in the first quadrant of the xy-plane according to the density $f(x, y) = \frac{4}{\pi}e^{-x^2-y^2}$. Let R be the distance from (X, Y) to the origin. Find the density of R.