Math 262

Section 4.5 Day 33

1. Simulate 10,000 averages, each of k samples from a Unif$[0, 1]$ distribution. Make a histogram of the 10,000 averages. Start with $k = 1$ and then try larger values of k. How does the shape of the histogram depend on k?

   Here is some code for making such a histogram in *Mathematica*:

   ```
   averages = Table[ Mean[ RandomVariate[ UniformDistribution[], k ] ], 10000 ]
   Histogram[averages]
   ```

   Here is similar code in R:

   ```
   averages <- replicate( 10000, mean(runif(k)) )
   hist(averages)
   ```

   Sketch the shape of your histograms:

2. Repeat the previous simulation, but now replace Unif$[0, 1]$ with a different distribution of your choice. What is the shape of the histogram? How does it depend on k?

3. Let $X_1, X_2, \ldots, X_{300}$ be iid random variables with mean μ_X and standard deviation σ_X. Also let $T = X_1 + X_2 + \cdots + X_{300}$ and $\overline{X} = \frac{T}{300}$.

   (a) What are the values of μ_T, σ_T, $\mu_{\overline{X}}$, and $\sigma_{\overline{X}}$?

   (b) What distributions are good approximations for T and $\overline{X}$?

4. Use the `Convolve` function in Mathematica to plot the pdf of $X_1 + X_2 + \cdots + X_n$, where each $X_i \sim \text{Unif}[0, 1]$ and $n \in \{1, 2, 3, 4, 5, 6\}$. Compare each pdf with the pdf of a normal distribution.

5. A farm packs tomatoes in crates. Individual tomatoes have mean weight of 10 ounces and standard deviation of 3 ounces. Estimate the probability that a crate of 40 tomatoes weighs between 380 and 410 ounces.