Math 262

Section 4.4 Day 32

1. Let X and Y have joint density $f(x, y) = \frac{1}{2}$ for $0 \le x \le y \le 2$.

   (a) Sketch the joint density of X and Y.

   (b) What is the marginal density of X?

   (c) Suppose you know that $X = \frac{2}{3}$. What does $f(\frac{2}{3}, y)$ tell you about the density of Y, given that $X = \frac{2}{3}$?

   (d) Suppose you know that $X = x_0$. What is then the density of Y?

   (e) In part (d), you found the conditional density $f_{Y|X}(y \mid x_0)$. How does this relate to the joint density $f(x, y)$ and the marginal density $f_X(x)$?

   (f) If $X = \frac{2}{3}$, then what is the probability that $Y \le 1$?

   (g) What is the expected value of Y given that $X = x_0$?

2. The joint pdf of X and Y is $f(x, y) = 3x$, for $0 \leq y \leq x \leq 1$.

   (a) What is the conditional distribution of X given $Y = y$?

   (b) What is $E(X \mid Y = y)$?

   (c) What is $\text{Var}(X \mid Y = y)$?

3. For continuous random variables X and Y, show that $E(E(X \mid Y)) = E(X)$.

4. The number of eggs N found in nests of a certain species of turtles has a Poisson distribution with mean λ. Each egg has probability p of being viable, and this event is independent from egg to egg. Find the mean and the variance of the number of viable eggs per nest.

★ **BONUS:** If X and Y are independent binomial random variables with identical parameters n and p, calculate the conditional expected value of X given that $X + Y = m$.