Math 262

Sections 4.1 and 4.2 Day 19

1. A cafeteria has three meal options: pizza, burgers, and salad bar. Three students each choose one option independently at random (equally likely to choose any option). Let X be the number (of the 3) who choose pizza, and let Y be the number who choose the salad bar.

   (a) What is the joint pmf of X and Y? What are the marginal pmfs of X and Y?

   (b) Are X and Y independent? Why or why not?

2. Suppose a particle is randomly located in the square $0 \leq x \leq 1$, $0 \leq y \leq 1$. That is, if two regions within the square have equal area, then the particle is equally likely to be in either region. Let (X, Y) be the coordinates of the particle.

   (a) What is the joint density function of X and Y?

   (b) Find $P(X \leq 0.2, 0.1 \leq Y \leq 0.5)$.

(c) Find $P(X \leq Y)$.

(d) Are X and Y independent? Why or why not?

3. Let X and Y have joint pdf $f(x, y) = 6xy^2$ for $0 \leq x \leq 1$ and $0 \leq y \leq 1$.

(a) Verify that $f(x, y)$ is a joint pdf.

(b) What is $P(X \leq Y)$?

(c) Find $f_X(x)$ and $f_Y(y)$.

(d) Are X and Y independent? Why or why not?

(e) Sketch the marginal pdfs $f_X(x)$ and $f_Y(y)$. What would you estimate to be the means $E(X)$ and $E(Y)$?

(f) Compute $E(X)$ and $E(Y)$.

(g) Compute $E(X+Y)$ in two different ways.

(h) Now compute $E(XY)$.

(i) What are the values of $\mathrm{Cov}(X,Y)$ and $\mathrm{Corr}(X,Y)$? (Try to do this without evaluating any more integrals.)

4. Let X and Y have joint pdf $f(x, y) = 3x + 3y$ for $0 \leq x$, $0 \leq y$, and $x + y \leq 1$.

   (a) Sketch the joint pdf and verify that the volume underneath is 1.

   (b) What values of X and Y are most likely? What values are not so likely?

   (c) Compute the following, using technology to evaluate integrals:

   - $E(X + Y)$
   - $E(XY)$
   - $E(X)$
   - $E(Y)$

   (d) What is $\text{Cov}(X, Y)$?