# Math 262

Section 3.7 Day 18

1. Let $X$ have density $f_X(x) = \frac{x}{2}$ for $0 \leq x \leq 2$, and let $Y = X^2$. What is the density of $Y$?

2. Let $X$ have density $f_X(x) = 2x$ for $0 \leq x \leq 1$, and let $Y = e^X$. What is the density of $Y$?

   (a) Sketch the transformation $y = e^x$ and identify the possible values of $Y$.

   (b) Find the cdf of $Y$, then differentiate to obtain the pdf.

   (c) Confirm that you obtain the same answer via the Transformation Theorem.

3. Let $X$ have density $f_X(x) = 2x$ for $0 \leq x \leq 1$, and let $Y = 2 - X^2$. What is the density of $Y$?

4. Let $X \sim N(0, 1)$ and $Y = X^2$. What is the distribution of $Y$?

5. Let $U \sim \text{Unif}[0, 1]$, and let $X$ have pdf $f(x)$.

   We wish to find a transformation from Unif$[0, 1]$ to the distribution of $X$. In other words, we want to find a function $g$ such that if $X = g(U)$, then the pdf of $X$ is $f(x)$.

   (a) If we want to apply the Transformation Theorem, what do we have to assume about $g$?

   (b) Apply the Transformation Theorem to the situation described above. How does the theorem allow you to find a transformation function $g$?

   (c) Does your function $g$ satisfy the assumptions of the Transformation Theorem? Explain.

6. Let $U \sim \text{Unif}[0,1]$, and let $X$ have pdf $f_X(x) = \begin{cases} x+1 & \text{if } -1 \le x \le 0, \\ 1-x & \text{if } 0 < x \le 1, \\ 0 & \text{otherwise.} \end{cases}$

We wish to find a transformation from $\text{Unif}[0,1]$ to the distribution of $X$.

(a) Sketch the pdf of $X$.

(b) Find a formula for the cdf $F_X(x)$. Also sketch $F_X(x)$.

(c) Sketch the inverse $F_X^{-1}(u)$. Then find a formula for $F_X^{-1}(u)$.

(d) Write a program to simulate values of $X$. (That is, generate values from $\text{Unif}[0,1]$, then apply $F_X^{-1}$ to each.) Simulate thousands of values and make a histogram. Does your histogram look like the density you sketched in part (a)?