Math 262

Sections 3.3 and 3.4 Day 15

1. Let Z be a standard normal random variable.

   (a) Draw a picture that represents $P(Z \leq 0.8)$. Then compute this probability.

   (b) Draw a picture that represents $P(Z \leq c) = 0.4$. Then find a number c that satisfies this equation.

2. Let X be a normal random variable with mean 24 and standard deviation 2.

   (a) Draw a picture that represents $P(23 \leq X \leq 25)$. Then compute this probability.

   (b) Draw a picture that represents $P(X \geq c) = 0.2$. Then find a number c that satisfies this equation.

3. What is the probability that a normal random variable is within 1.5 standard deviations of its mean?

4. Suppose that a fair, 6-sided die is rolled 1000 times. Use a normal distribution to approximate the probability that the number 6 appears between 150 and 200 times (inclusive).

★ **BONUS:** Let $f(x)$ denote the pdf of the $N(\mu, \sigma)$ distribution. Show that the points of inflection lie at $x = \mu \pm \sigma$. (*Hint*: differentiate twice with respect to x.)

5. Suppose that emails arrive in your inbox according to a Poisson process with rate 2 emails per hour. Then the time between successive emails is an exponential random variable with mean 30 minutes.

   (a) What is the probability that an email arrives in the next 20 minutes?

   (b) What is the probability that you don't receive any emails in the next hour?

   (c) What is the standard deviation of the time until the next email?

6. Let $X \sim \text{Exp}(\lambda)$ and $0 < a < b$.

   (a) What is $P(X \geq a)$?

   (b) Show that $P(X > b \mid X > a) = P(X > b - a)$.

   (c) What other distribution satisfies the equality in (b)?

   (d) The property in (b) is special, in the sense that it doesn't hold for most random variables. For example, if $U \sim \text{Unif}[0, 10]$, show that $P(U > 4 \mid U > 3) \neq P(U > 1)$.

7. What is the moment generating function of an exponential random variable?

8. Let $X \sim \text{Exp}(1)$. Find a formula for $E(X^n)$ for positive integers n.