Math 262

Review for Exam 1 Day 9

1. Suppose you roll five (fair, 6-sided) dice. Define the following events:
   A: exactly four of the five dice show the value 1
   B: exactly three of the five dice show the value 1
   C: exactly two of the five dice show the value 1
   D: the sum of the values on the five dice is 8

   (a) What is $P(A)$?

   (b) What is $P(A \cup B \cup C)$?

   (c) What is $P(A \mid D)$?

2. Consider a 20-sided die (an icosahedron) with values from 1 to 20. Roll the die one time. Let A denote the event that the value is even. Let B denote the event that the value is 13 or higher.

   (a) Are A and B disjoint events? Why?

   (b) Are A and B independent events? Why?

   (c) Calculate $P(A \cup B)$ using the inclusion-exclusion formula.

3. An urn contains 10 balls: 4 red and 6 blue. A second urn contains 16 red balls and an unknown number of blue balls. A single ball is drawn from each urn. The probability that both balls are the same color is 0.44. How many blue balls are in the second urn?[1]

[1] Actuary Exam P practice problem

4. Let X be the amount of time until the next message appears on your social media feed. Suppose $E(X) = 26$ seconds and $\sigma_X = 4$ seconds.

   (a) Find a lower bound on the probability that X is between 20 and 32 seconds.

   (b) Find an interval that contains X with a probability of at least 0.9.

5. An urn contains 5 red, 6 blue, and 8 green balls. If a set of 3 balls is randomly selected, what is the probability that each of the balls will be (a) of the same color; (b) of different colors? Repeat under the assumption that the balls are sampled with replacement: whenever a ball is selected, its color is noted and it is replaced in the urn before the next selection. (*Hint*: When sampling with replacement, each *ordered* selection is equally likely.)

6. A roulette wheel has 12 numbers colored red (R) or black (B) as follows:

1	2	3	4	5	6	7	8	9	10	11	12
R	R	B	R	B	B	B	B	R	B	R	R

   Let A be the event that a spin of the wheel yields an red number. Let B be the event that a spin of the wheel yields an even number. Let C be the event that a spin of the wheel yields a number less than 7. Are events A, B, and C (pairwise) independent? Are they mutually independent?