The next two problems require **Chebyshev's Inequality**: Let X be a discrete random variable with mean μ and standard deviation σ. For any $k \geq 1$,

$$P\left(|X - \mu| \geq k\sigma\right) \leq \frac{1}{k^2}.$$

In words, *the probability that X is at least k standard deviations away from its mean is at most $\frac{1}{k^2}$.*

1. Verify that Chebyshev's Inequality holds with $k = 2$ for the random variable X from Problem 4 from the previous class. That is, check that $P\left(|X - \mu| \geq 2\sigma\right)$ is less than $\frac{1}{(2)^2}$.

2. The number of equipment breakdowns in a manufacturing plant averages 4 per week, with a standard deviation of 0.7 per week.

   (a) Find an interval that includes at least 90% of the weekly figures for the number of breakdowns.

   (b) A plant supervisor promises that the number of breakdowns will rarely exceed 7 in a one-week period. Is the supervisor justified in making this claim? Why?

3. Suppose that 45% of the phone calls you receive are scam calls. Assume that the probability of a scam call is independent from one call to the next.

   (a) Let $X = 1$ if the next call you receive is a scam call, and let $X = 0$ otherwise. What type of random variable is X? What are its mean and standard deviation?

   (b) Let Y be the number of scam calls in the next 40 phone calls. What type of random variable is Y? Sketch the pmf of Y.

   (c) What are the mean and standard deviation of Y?

   (d) Suppose that you lose 30 seconds of your time every time a scammer calls your phone. What is the expected value and standard deviation of the amount of time you will lose over the next 40 phone calls?

4. A coin that lands on heads with probability p is flipped ten times. Given that a total of 6 heads results, what is the conditional probability that the first three flips are *heads*, *tails*, *heads* (in that order)?

5. Among persons donating blood to a clinic, 85% have Rh^+ blood. Six people donate blood at the clinic on a particular day.

   (a) Find the probability that at most three of the six have Rh^+ blood.

   (b) Find the probability that at most one of the six does not have Rh^+ blood.

   (c) What is the probability that the number of Rh^+ donors lies within two standard deviations of the mean number?

   (d) The clinic needs six Rh^+ donors on a certain day. How many people must donate blood to have the probability of obtaining blood from at least six Rh^+ donors over 0.95?

★ **BONUS:** Let $X \sim \text{Bin}(n, p)$. Show that $E(X) = np$.

Hint: Write $E(X)$ as a sum and factor out np. Then use the binomial theorem to show that the sum equals 1.

★ **BONUS:** A system consists of n components, each of which will independently function with probability p. The system will operate effectively if at least one-half of its components function. For what values of p is a 5-component system more likely to operate effectively than a 3-component system?