# Math 262

Sections 2.1–2.3 Day 6

1. The cdf for a random variable $X$ is as follows:

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.2 & 0 \le x < 1 \\ 0.5 & 1 \le x < 2 \\ 0.8 & 2 \le x < 4 \\ 1 & 4 \le x \end{cases}$$

(a) What is $P(X = 2)$?

(b) What is $P(X = 3)$?

(c) What is $P(2.5 \le X)$?

(d) Sketch the pmf of $X$.

2. Each of the following functions might be the pmf for some random variable $X$. *Discuss with your group*: How can you determine whether a given function is a pmf? Which of these functions is a pmf?

   (a) $p(x) = 2 - 3x$ for $x \in \{0, 1\}$

   (b) $p(x) = \frac{x^2}{50}$ for $x \in \{1, 2, \ldots, 5\}$

   (c) $p(x) = \log_{10}\left(\frac{x+1}{x}\right)$ for $x \in \{1, 2, \ldots, 9\}$

3. Which of the following properties must hold for any cdf $F(x)$? Discuss each property with your group. Either say why it must hold or give a counterexample to show that it might not hold.

   (a) $\lim_{b \to -\infty} F(b) = 0$

   (b) $\lim_{b \to \infty} F(b) = 1$

   (c) $F(x)$ is continuous

   (d) $F(x)$ is nondecreasing; that is, if $a < b$, then $F(a) \leq F(b)$

   (e) $F(b) = 0.5$ for some value $b$

4. Let $X$ be a random variable with pmf given by $p(4) = 0.3$, $p(5) = 0.2$, $p(8) = 0.3$, and $p(10) = 0.2$.

   (a) What is the expected value $E(X)$?

   (b) What is $E(X^2)$?

   (c) What is $\text{Var}(X)$? *Hint: use the shortcut formula!*

   (d) Suppose the random variable is part of a game in which you win $2X - 8$ dollars. Let $Y = 2X - 8$. What is the pmf of $Y$?

   (e) Use the pmf of $Y$ to find $E(Y)$, your expected winnings in this game.

   (f) Use the pmf of $Y$ to find $E(Y^2)$, and then find is $\text{Var}(Y)$.

   (g) How is $E(Y)$ related to $E(X)$? How is $\text{Var}(Y)$ related to $\text{Var}(X)$?

★ **BONUS:** Three balls are randomly selected (without replacement) from an urn containing 20 balls numbered 1 through 20. Let random variable $X$ be the largest of the three selected numbers. What is $P(X = 17)$? What is $P(X \geq 17)$?