Math 262

1. The probability that Prisca studies for a test is 0.8. The probability that she studies and passes the test is 0.7. If Prisca studies, what is the probability that she passes the test?

2. A machine produces parts, 10% of which are defective. An inspector is able to remove 95% of the defective parts. What is the probability that a part is defective *and* removed by the inspector?

3. A soccer team wins 60% of its games when it scores the first goal, and 30% of its games when the opposing team scores first. If the team scores first in 40% of its games, what percent of its games does it win?
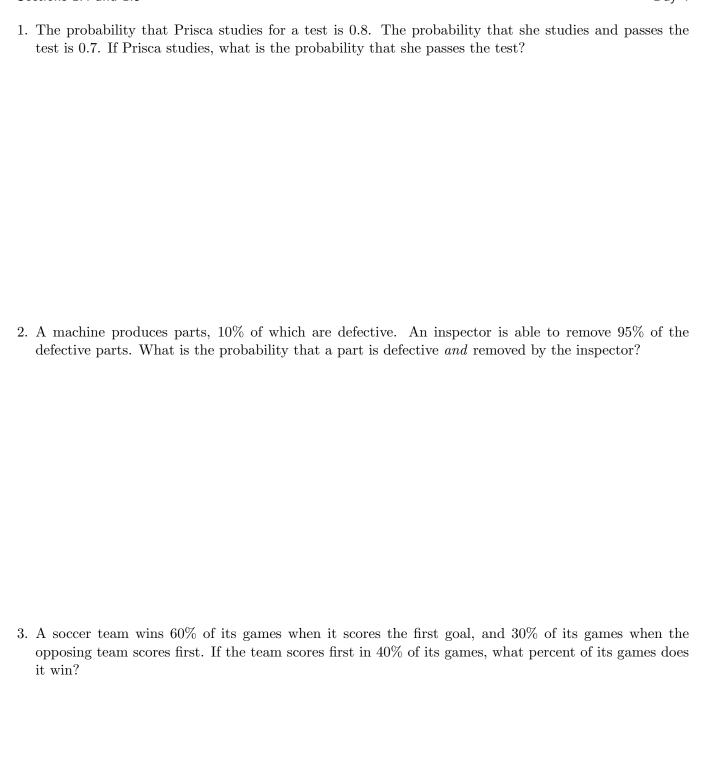
4. A factory uses 3 machines to produce certain items. Machine A produces 50% of the items, 6% of which are defective. Machine B produces 30% of the items, 4% of which are defective. Machine C produces 20% of the items, 3% of which are defective.

   (a) What is the probability that a randomly-selected item is defective?

   (b) If an item is defective, what is the probability that it was produced by Machine A?

5. Suppose that a patient is tested for a disease. Let A be the event that the test is positive, and let D be the event that the patient actually has the disease. Further suppose that:

   $P(A \mid D) = 0.99$ (*sensitivity*: probability of a positive test if the patient has the disease)

   $P(A' \mid D') = 0.99$ (*specificity*: probability of a negative test if the patient doesn't have the disease)

   (a) *Rare Disease:* If $P(D) = 0.01$, what is the probability that a patient who tests positive actually has the disease?

   (b) *Common Disease:* If $P(D) = 0.1$, what is the probability that a patient who tests positive actually has the disease?

6. A red die and a blue die are rolled. Let A be the event that the red die rolls 2, let B be the event that the sum of the rolls is 5, and let C be the event that the sum of the rolls is 7. Are A and B independent events? How about A and C?

7. A sequence of n independent trials are to be performed. Each trial results in a success with probability p and a failure with probability $1 - p$. What is the probability that...

   (a) ...all trials result in successes?

   (b) ...at least one trial results in a success?

   (c) ...exactly k trials result in successes?

8. If A and B are independent events each with positive probability, show that they cannot be mutually exclusive.

★ **BONUS:** Box 1 contains 5 red balls and box 2 contains 5 blue balls. Balls are randomly removed in the following manner: after each removal from box 1, a ball is taken from box 2 (if box 2 has any balls) and placed in box 1. This process continues until all balls have been removed (so ten removals total). What is the probability that the final ball removed from box 1 is red?