Math 262

More counting methods Day 3

1. In how many ways can 12 distinct books be distributed among four (distinct) children so that

   (a) Each child receives three books?

   (b) The two oldest children receive four books each, while the two youngest children receive two books each?

2. How many ways can you place 9 identical balls in 4 different boxes?

3. How many different dominoes can be formed with the numbers $1, 2, \ldots, 6$? How about if the numbers $1, 2, \ldots, 12$ are used?

4. How many ways can 7 identical jobs be assigned to 10 (distinct) people...

   (a) ...if no person can do multiple jobs?

   (b) ...if a single person can do multiple jobs?

5. Seven awards are to be distributed to 10 (distinguishable!) mathletes. How many different distributions are possible if:

   (a) The awards are identical and nobody gets more than one?

   (b) The awards are different and nobody gets more than one?

   (c) Awards are identical and anyone can get any number of awards?

6. Consider the 20 "integer lattice points" (a, b) in the xy-plane given by $0 \leq a \leq 4$ and $0 \leq b \leq 3$, with a and b integers. (Draw a little picture.) Suppose you want to walk along the lattice points from $(0, 0)$ to $(4, 3)$, and the only legal steps are one unit to the *right* or one unit *up*.

   (a) How many legal paths are there from $(0, 0)$ to $(4, 3)$?

   (b) How many legal paths from $(0, 0)$ to $(4, 3)$ go through the point $(2, 2)$?

7. A box contains 5 red, 6 white, and 7 blue balls. The box is stirred and five balls are chosen without replacement. What is the probability that the 5 balls chosen include at least one of each color? Do this in steps:

   (a) Let E_1 be the event that *no red ball* is chosen, E_2 the event that *no white ball* is chosen, and E_3 the event that *no blue ball* is chosen. Find the probabilities $P(E_1)$, $P(E_2)$, and $P(E_3)$.

   (b) Find the probabilities $P(E_1 \cap E_2)$, $P(E_1 \cap E_3)$, $P(E_2 \cap E_3)$, and $P(E_1 \cap E_2 \cap E_3)$.

   (c) Use the inclusion-exclusion principle to find $P(E_1 \cup E_2 \cup E_3)$.

   (d) Use the preceding result to answer the original question.

8. Determine how many nonnegative integer solutions satisfy the equation

$$x_1 + x_2 + x_3 + x_4 = 7.$$

For example, one solution is $x_1 = x_2 = 1$, $x_3 = 0$, $x_4 = 5$, which is different from the solution $x_1 = 1$, $x_2 = 0$, $x_3 = 1$, $x_4 = 5$.

First rephrase this problem as a selection problem. Is selection with or without replacement? Does order matter?

★ **BONUS:** These are fun, but a bit more complicated than the previous problems.

(a) How many ways can 24 students be divided into 4 groups of equal size?

(b) What is the probability that a randomly chosen arrangement of the letters in *MISSISSIPPI* contains 4 consecutive *I*s?