Math 262

Sections 1.2 and 1.3 Day 2

1. Write down probability Axiom 3. Let $A_i = \varnothing$ for all $i \in \{1, 2, 3, \ldots\}$. At your table, explain why this implies $P(\varnothing) = 0$.

2. The **Complement Rule** says that for any event A, $P(A) = 1 - P(A')$. (This can be proved using Axiom 3.) Explain how the Complement Rule implies that $P(A) \leq 1$ for any event A.

3. If A and B are disjoint, Axiom 3 implies that $P(A \cup B) = P(A) + P(B)$. If A and B are *not* disjoint, what is the relationship between $P(A \cup B)$, $P(A \cap B)$, $P(A)$, and $P(B)$?

   Hint: Use a Venn diagram.

4. Generalize your answer from #3 to three sets. That is, what can you say about $P(A \cup B \cup C)$?

★ **BONUS:** If $A \subseteq B$ (meaning that A is a subset of B), show that $P(A) \leq P(B)$.

5. A painter has six cans of paint, each containing a different color. Two of the cans contain paint with a flat finish and four of the cans contain glossy paint.

   (a) If the painter selects one can of flat paint and one can of glossy paint, how many different color combinations are possible? How does this relate to the Fundamental Counting Principle?

   (b) Suppose the painter forgets that the cans contain paint with different finish, and simply selects two cans at random. Use a tree diagram to help you find the probability that the two selected cans have the *same* finish.

6. Minnesota issues license plates that consist of three numbers followed by three letters; for example: 012-ABC. How many different license plates of this form are possible?

7. How many different 4-letter codes can be made from the letters in the word *PADLOCKS*, if no letter can be chosen more than once? How about 6-letter codes from the letters in *DOGWATCHES*?

8. In a certain lottery, players select six numbers from 1 to n. For each drawing, balls numbered 1 to n are placed in a hopper, and six balls are drawn at random and without replacement. To win, a player's numbers must match those on the balls, in any order.

   (a) If $n = 15$, how many combinations of winning numbers are possible?

   (b) If $n = 24$, how many combinations of winning numbers are possible?

   (c) If $n = 24$, what is the probability that the six balls that are drawn contain only numbers less than 16?

   (d) If $n = 24$, what is the probability that the ball numbered 8 is among the balls drawn?

9. An absent-minded secretary prepared five letters and envelopes addressed to five different people. Then the secretary placed the letters randomly in the envelopes. A match occurs if a letter and its envelope are addressed to the same person. What is the probability of the following events?

   (a) All five letters and envelopes match.

   (b) Exactly four of the five letters and envelopes match.

   (c) **BONUS:** None of the letters and envelopes match.