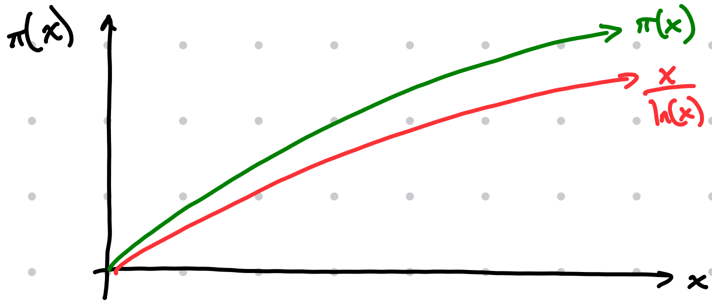
MATH 242 — 13 April 2026

# How are the primes distributed?

## What is the shape of the prime-counting function?

$\pi(x)$: gives the number of primes up to $x$




# DENSITY OF PRIMES


$$\text{approx density of primes near } x = \frac{\text{count of primes in the window}}{\text{length of this window}} = \frac{\pi(x + \text{length}) - \pi(x)}{\text{length}}$$

If the density of primes near $x$ is $\frac{1}{\ln(x)}$, then the count of primes up to $x$ is...

$$\pi(x) = \int_0^x \frac{1}{\ln(t)}\,dt$$

logarithmic integral

It looks like $li(x) > \pi(x)$, though these quantities are relatively close for large $x$

Skewes proved: $\pi(x) > li(x)$ for some $x$ satisfying

$$x < 10^{10^{10^{34}}}$$

Recent estimates: $x \approx 1.397 \times 10^{316}$

# The RIEMANN ZETA FUNCTION

$$\zeta(s) = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

zeta

$$\zeta(2) = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$$