MATH 242 — 10 April 2026

THEOREM: There are infinitely many primes.

proof: Suppose not. Suppose the only primes are

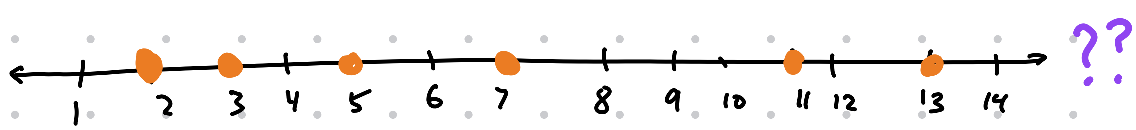
$$p_1, p_2, p_3, \dots, p_N.$$

Consider: $M = p_1 p_2 p_3 \cdots p_N + 1$

- Then M is not prime, since it's bigger than all the primes in the list.
- But M cannot be composite, since its 1 more than the product of all primes

Contradiction. So there are infinitely many primes.

How are the primes distributed?



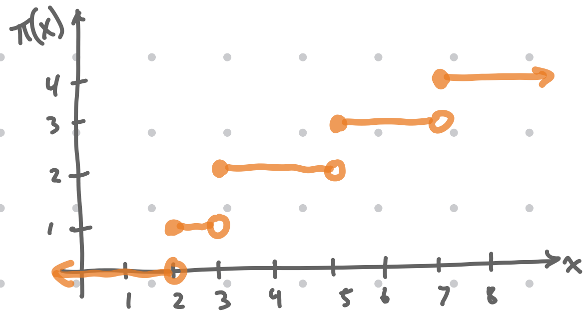
Given any positive number x, how many primes are less than or equal to x?

Definition: The prime counting function $\pi(x)$ gives the number of primes less than or equal to x.

$$\pi(x) : \mathbb{R} \to \mathbb{Z}^{\geq 0}$$

examples: $\pi(10) = 4$ $\pi(1) = 0$ $\pi(12.3) = 5$

Goal: Make a plot of $\pi(x)$



What is the shape of the prime counting function?

Efficiently computing $\pi(x)$ for many values of x:

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primes: [2, 3, 5, 7, 11, 13, 17, 19, ...]

i= ...	0	1	2	3	4	5	6	7		nMax+1
counts:	[0,	0,	1,	2,	2,	3,	0,	0,	...,	nMax]
	$\pi(0)$	$\pi(1)$								$\pi(\text{nMax})$