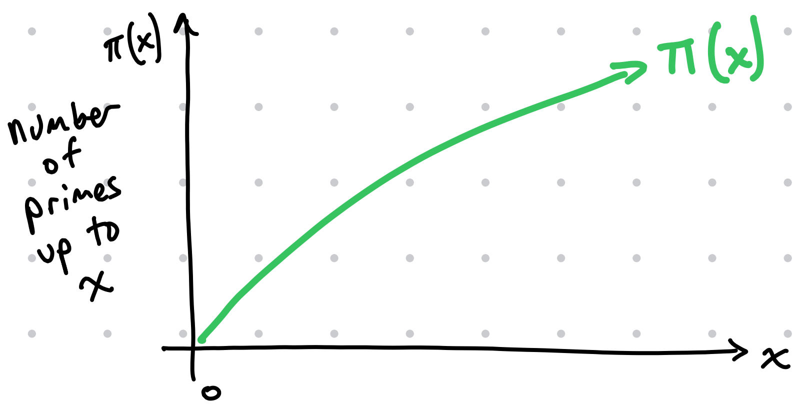
15 April 2024

How are the primes distributed? What is the shape of the prime counting function?



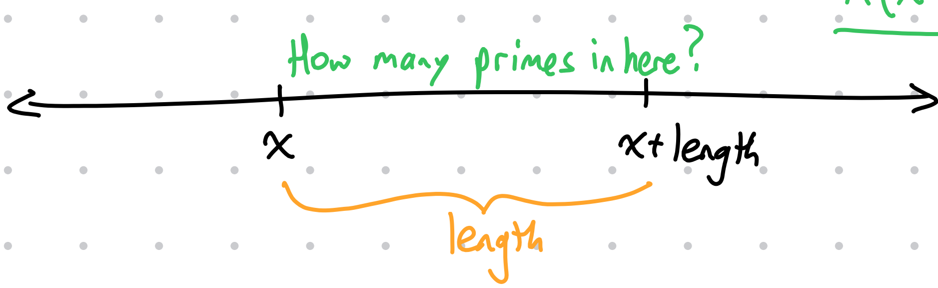
PRIME NUMBER THEOREM

$$\lim_{x \to \infty} \frac{\pi(x)}{\frac{x}{\ln(x)}} = 1 \quad \text{and} \quad \lim_{x \to \infty} \frac{\pi(x)}{\mathrm{li}(x)} = 1$$

Density of primes near x:

approx. density:

$$\frac{\pi(x + \text{length}) - \pi(x)}{\text{length}}$$



Gauss: density of primes near x is approx. $\frac{1}{\ln(x)}$

If so, then the number of primes up to x is approx. $\boxed{\int_0^x \frac{1}{\ln(t)}\,dt} = li(x)$

logarithmic integral

It appears that $\pi(x) < li(x)$.

However, it is proved that $\pi(x) - li(x)$ changes sign infinitely many times, but the precise x for which this occurs is unknown.

<u>estimates:</u> $x < 10^{10^{10^{34}}}$

$x \approx 1.397 \times 10^{316}$

Riemann Zeta Function

$$\zeta(s) = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n^s}$$