# Applications of Percolation Theory

Math 242 Modern Computational Math

# Groundwater Hydrology

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Percolation Theory and Its Application to Groundwater Hydrology

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*Racah Institute of Physics, H...*

The theory of percolation, originally propos...
porous media, has undergone enormous develo...
The principal advantage of percolation theory ...
geometrical and physical properties of the syst...
percolation theory to date, and identifies aspect...
methods of percolation theory are discussed, ...
problems are reviewed, and future directions f...

1. INTRODUCTION

The theory of percolation was first introduced some tin...
ago to describe polymerization [*Flory*, 1941; *Stockmaye*...
1943] and penetration of fluids in porous media [*Broadbe*...
*and Hammersley*, 1957], and the subject has since be...
intensively studied, primarily in the field of physics [e.g...
*Stauffer*, 1985; *Balberg*, 1987, and references therein]. Ho...
ever, little direct use of percolation theory results has be...
made to date in the field of hydrology [*Thompson et al.*,
1987]. The theory has been extensively developed as a ...

... ogy are then briefly surveyed, followed by a review of
existing and potential applications of percolation theory to

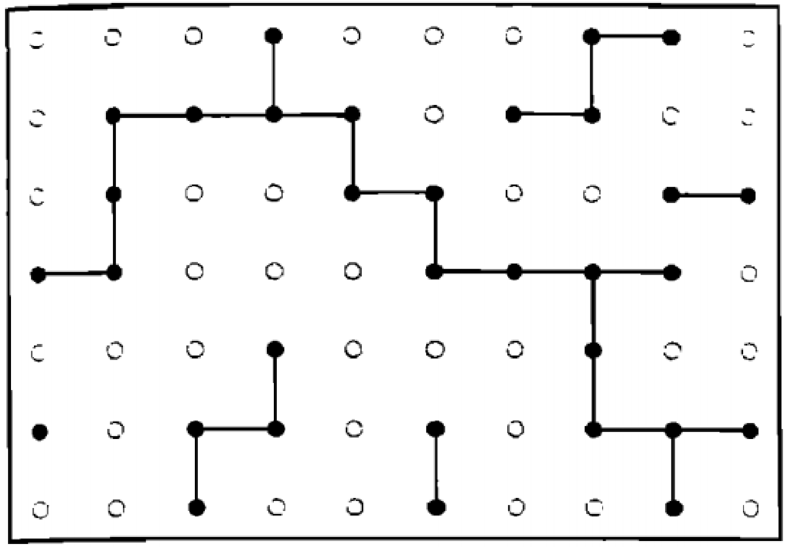

Fig. 3. An illustration of a finite lattice with various size clusters. There is one cluster, the percolation cluster, that connects the bottom and top boundaries, and the left and right boundaries, of the finite square lattice.

# Rock Fractures and Earthquakes

Z. L. Wu, Implications of a percolation model for earthquake ‘nucleation’, *Geophysical Journal International*, Volume 133, Issue 1, April 1998, Pages 104–110, https://doi.org/10.1046/j.1365-246X.1998.1331491.x

*Geophys. J. Int.* (1998) **133,** 104–110

## Implications of a percolation model for earthquake ‘nucleation’

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SUMMARY

A percolation model is applied to the explanation of some of the qualitative and quantitative aspects associated with the recent observations of earthquake ‘nucleation’. An additional assumption is introduced that nucleation starts at the critical point of percolation. The model explains the order of magnitude of the seismic moment release during the nucleation, the dependence of the seismic moment of the main shock on the duration of the nucleation process, and the observation that the fraction of the moment release during the nucleation has no systematic variation with the size of the main shock. The model also suggests that the source time function of the nucleation phase may be complex, and also that not all earthquakes are accompanied by a nucleation process, which is supported by observational results. By assuming that there exists a scale invariance associated with the criticality, a Widom scaling model is proposed to describe the electromagnetic emission during earthquake rupture.

**Key words:** earthquake rupture, electromagnetic emission, nucleation, percolation.

### 1 INTRODUCTION

In recent years, the study of the nucleation of earthquake

### 2 PERCOLATION MODEL

There are different ways of representing the problem of earth-

# Electrical conductivity of composite materials

Eletskii, Knizhnik, Potapkin, and Kenny, “Electrical characteristics of carbon nanotube-doped composites”, Physics-Uspekhi, 58 (3) 2015, 209-251, doi:10.3367/ufne.0185.201503a.0225.

*Physics – Uspekhi* **58** (3) 209 – 251 (2015)


*REVIEWS OF TOPICAL PROBLEMS*

PACS numbers: 72.80.Tm, 73.61.Ph, 73.63.Fg

## Electrical characteristics of carbon nanotube-doped composites

A V Eletskii, A A Knizhnik, B V Potapkin, J M Kenny

DOI: 10.3367/UFNe.0185.201503a.0225

### Contents



**Abstract.** **This paper reviews research into the electrical properties that are imparted to composite materials by introducing carbon nanotubes (CNTs) into their polymer matrices. Due to the large aspect ratio of CNTs, even a small amount of doping (at a level of 0.01 – 0.1%) is enough to increase the conductivity of the material by more than ten orders of magnitude, thus changing it from an insulator to a conductor. At low doping, charge transfer is of a percolation nature in the sense that nanotubes that are in contact with each other form conducting channels in the material. Importantly, the conductivity has a threshold nature, so that the conduction jump occurs upon an arbitrarily small increase in a doping level above the critical value. This paper summarizes experimental data on the position of the percolation threshold and on the conductivity of the composites above the threshold and discusses the methods for and basic results obtained from the simulation of the percolation conductivity of CNT-doped composites are discussed. Particular attention is given to contact phenomena that occur at adjacent nanotube boundaries and which determine the conductivity of CNT-doped composites.**

**Keywords:** polymers, composites, carbon nanotubes, electrical properties

### 1. Introduction

Polymer materials are finding applications in many technological processes and engineering systems due to their good

# Battery Chemistry

Glazier, Li, Zhou, Bond, Dahn. "Characterization of Disordered Li(1+x)Ti2xFe(1–3x)O2 as Positive Electrode Materials in Li-Ion Batteries Using Percolation Theory." *Chemistry of Materials*, 2015, *27* (22), 7751-7756 doi: 10.1021/acs.chemmater.5b03530.

CHEMISTRY OF MATERIALS
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## Characterization of Disordered $Li_{(1+x)}Ti_{2x}Fe_{(1-3x)}O_2$ as Positive Electrode Materials in Li-Ion Batteries Using Percolation Theory

Stephen L. Glazier,[†] Jing Li,[‡] Jigang Zhou,[§] Toby Bond,[§] and J. R. Dahn*[,†,‡,‖]

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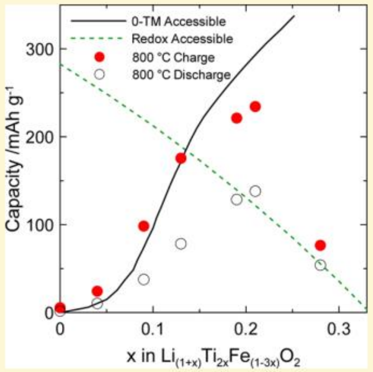
ABSTRACT: Recent theoretical and experimental works have shown that disordered positive electrode materials can function well in lithium cells. This work explores the solid solution series $Li_{(1+x)}Ti_{2x}Fe_{(1-3x)}O_2$ ($0 \leq x \leq 0.333$) and compares the measured specific capacity variation with $x$ to a recent theoretical model. The samples have varying degrees of cation disordering between lithium and transition metal layers that is dependent on $x$. The materials were characterized using induced coupled plasma optical emission spectroscopy, scanning electron microscopy, X-ray diffraction, and X-ray absorption spectroscopy (XAS) to quantify the degree of disorder and predict electrochemical performance. The specific capacities of lithium-limited samples ($0 \leq x \leq 0.13$) were found to agree very well with the recently proposed percolation theory model, whereas redox-limited samples ($0.13 \leq x \leq 0.29$) yielded slightly higher than expected capacities due to oxygen redox compensation characterized by oxygen K-edge XAS studies. Capacity retention was found to increase with lithium content. The voltage vs specific capacity relations for this set of materials do not suggest practicality, so this work is primarily of academic interest, but it suggests that more disordered materials should be explored.



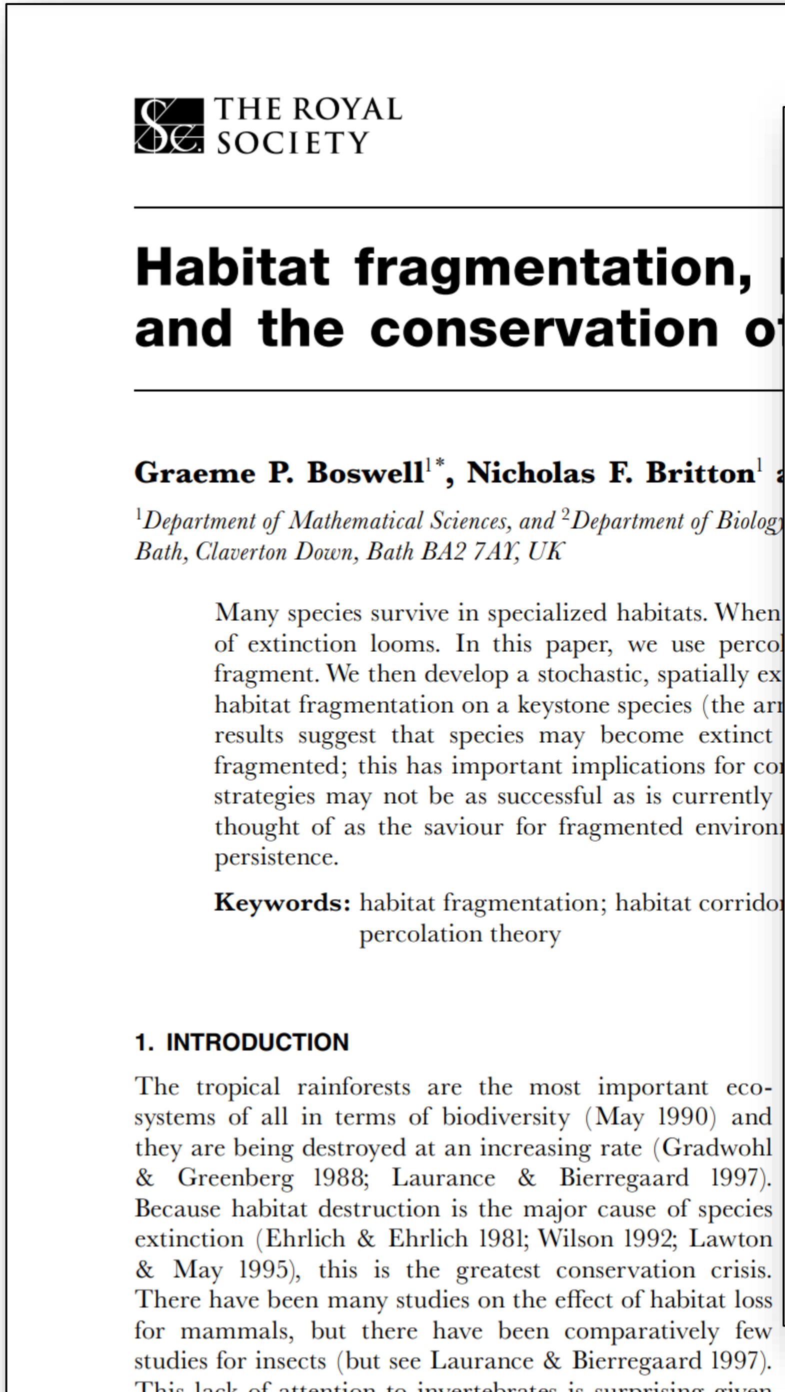

### INTRODUCTION

Large-scale lithium-ion batteries are becoming more in demand for use in automotive and energy storage applications. In order to fulfill these demands, new materials must be cheaper and have high energy density, long life, and good safety. New positive electrode materials are, therefore, of interest for their

to in the literature, channels, defined by Lee et al. as $n$-TM, where $n$ is the number of transition metals around the tetrahedral sites ($n$ = 0–4).[4] The word channel may misrepresent the geometry and path of Li through these tetrahedra; therefore, the term site will be used in this work in place of channel. In traditional ordered-layered structures, like $LiCoO_2$, only 1-TM and 3-TM sites are present and Li migrates

# Animal Habitats

Boswell, Britton, and Franks, “Habitat fragmentation, percolation theory, and the conservation of a keystone species”, *Proceedings of the Royal Society B: Biological Sciences* vol. 265,1409 (1998): 1921–1925. doi:10.1098/rspb.1998.0521.



THE ROYAL SOCIETY

**Habitat fragmentation, … and the conservation o…**

**Graeme P. Boswell**[1*], **Nicholas F. Britton**[1] …

[1]*Department of Mathematical Sciences, and* [2]*Department of Biolog…*
*Bath, Claverton Down, Bath BA2 7AY, UK*

Many species survive in specialized habitats. When … of extinction looms. In this paper, we use perco… fragment. We then develop a stochastic, spatially ex… habitat fragmentation on a keystone species (the ar… results suggest that species may become extinct … fragmented; this has important implications for co… strategies may not be as successful as is currently … thought of as the saviour for fragmented environ… persistence.

**Keywords:** habitat fragmentation; habitat corrido… percolation theory

**1. INTRODUCTION**

The tropical rainforests are the most important ecosystems of all in terms of biodiversity (May 1990) and they are being destroyed at an increasing rate (Gradwohl & Greenberg 1988; Laurance & Bierregaard 1997). Because habitat destruction is the major cause of species extinction (Ehrlich & Ehrlich 1981; Wilson 1992; Lawton & May 1995), this is the greatest conservation crisis. There have been many studies on the effect of habitat loss for mammals, but there have been comparatively few studies for insects (but see Laurance & Bierregaard 1997). This lack of attention to invertebrates is surprising given …

model strongly suggests that there are critical thresholds both in the size of habitat islands and in the number of degraded patches within such habitat islands, beyond …

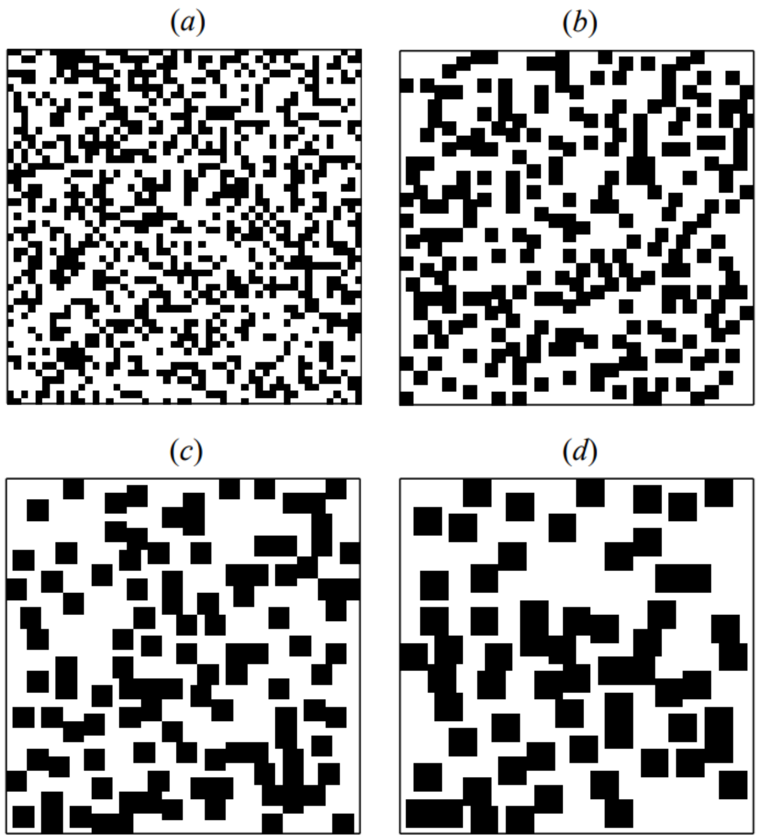
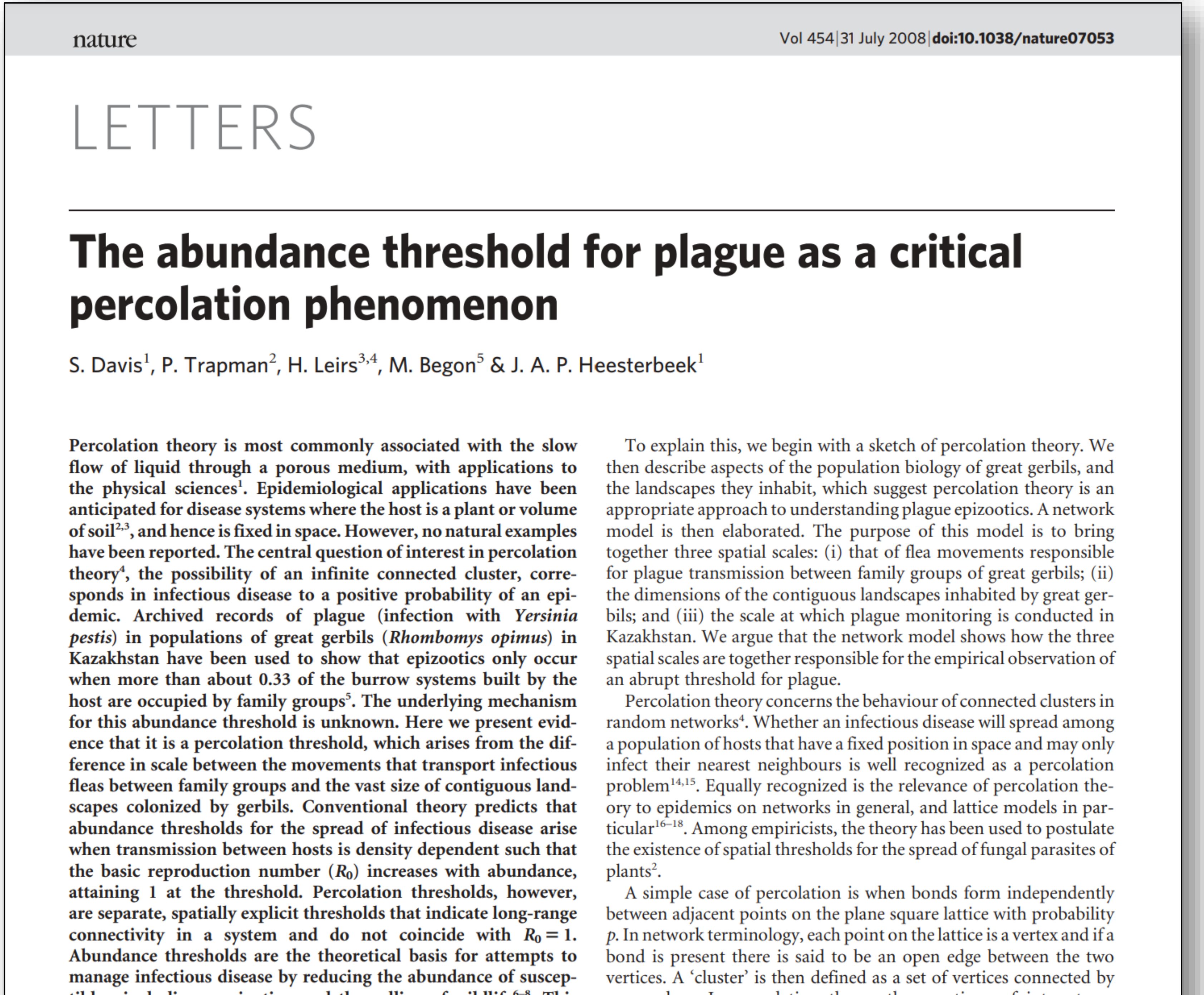



Figure 5. The effect of ‘randomly’ removing 40% of the habitat from a $50 \times 50$ lattice. The black patches represent the removed habitat; the white patches represent forested areas. (*a*) The patches are removed totally at random. (*b*) The patches are removed in blocks of four patches (i.e. $2 \times 2$). (*c*) The patches are removed in blocks of $3 \times 3$. (*d*) Patches removed in blocks of $4 \times 4$.

# Spread of Disease

Davis, Trapman, Leirs, *et al.* The abundance threshold for plague as a critical percolation phenomenon. *Nature* 454, 634–637 (2008). https://doi.org/10.1038/nature07053.



nature Vol 454|31 July 2008|**doi:10.1038/nature07053**

LETTERS

## The abundance threshold for plague as a critical percolation phenomenon

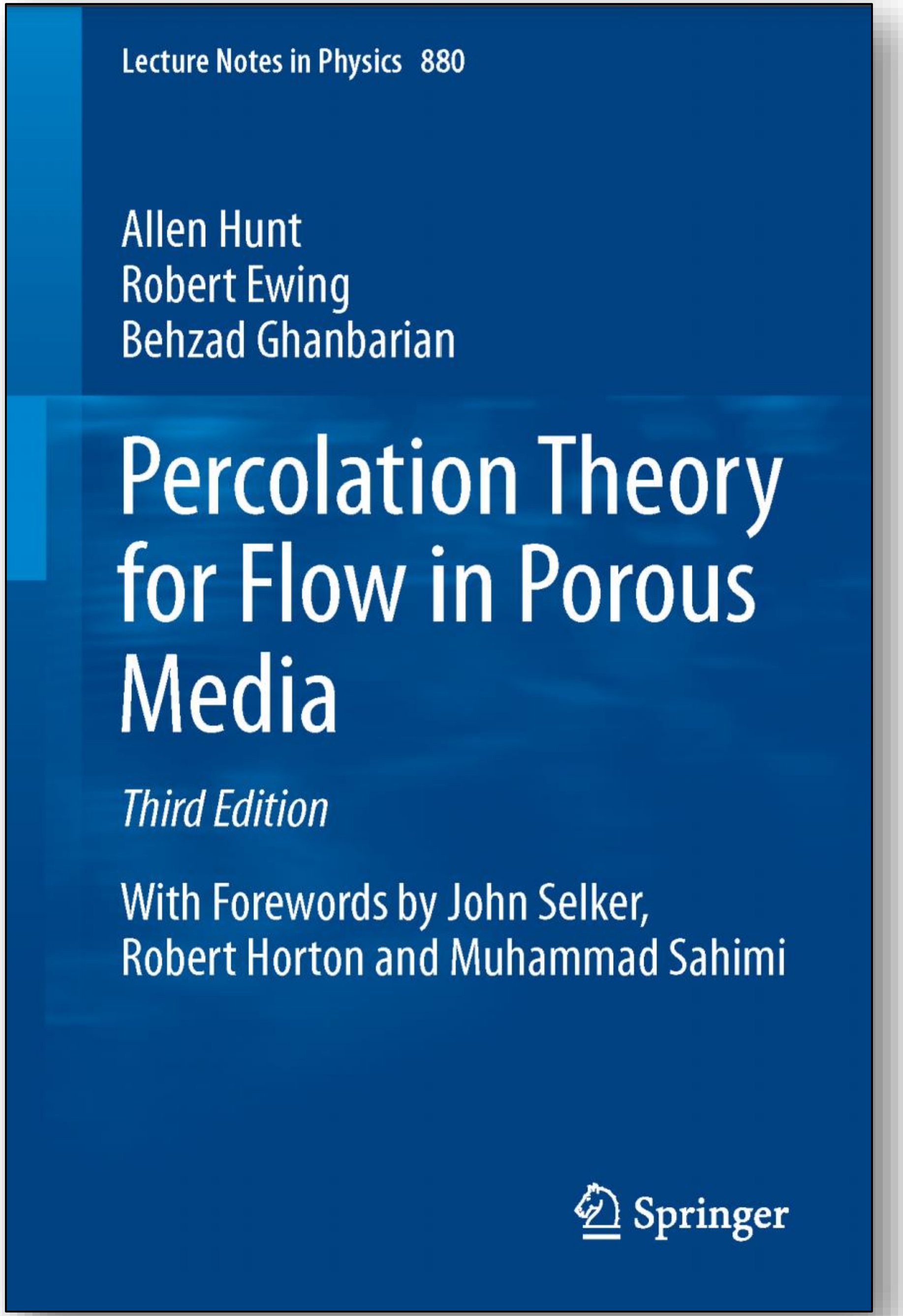
S. Davis[1], P. Trapman[2], H. Leirs[3,4], M. Begon[5] & J. A. P. Heesterbeek[1]

**Percolation theory is most commonly associated with the slow flow of liquid through a porous medium, with applications to the physical sciences[1]. Epidemiological applications have been anticipated for disease systems where the host is a plant or volume of soil[2,3], and hence is fixed in space. However, no natural examples have been reported. The central question of interest in percolation theory[4], the possibility of an infinite connected cluster, corresponds in infectious disease to a positive probability of an epidemic. Archived records of plague (infection with *Yersinia pestis*) in populations of great gerbils (*Rhombomys opimus*) in Kazakhstan have been used to show that epizootics only occur when more than about 0.33 of the burrow systems built by the host are occupied by family groups[5]. The underlying mechanism for this abundance threshold is unknown. Here we present evidence that it is a percolation threshold, which arises from the difference in scale between the movements that transport infectious fleas between family groups and the vast size of contiguous landscapes colonized by gerbils. Conventional theory predicts that abundance thresholds for the spread of infectious disease arise when transmission between hosts is density dependent such that the basic reproduction number ($R_0$) increases with abundance, attaining 1 at the threshold. Percolation thresholds, however, are separate, spatially explicit thresholds that indicate long-range connectivity in a system and do not coincide with $R_0 = 1$. Abundance thresholds are the theoretical basis for attempts to manage infectious disease by reducing the abundance of suscep-**

To explain this, we begin with a sketch of percolation theory. We then describe aspects of the population biology of great gerbils, and the landscapes they inhabit, which suggest percolation theory is an appropriate approach to understanding plague epizootics. A network model is then elaborated. The purpose of this model is to bring together three spatial scales: (i) that of flea movements responsible for plague transmission between family groups of great gerbils; (ii) the dimensions of the contiguous landscapes inhabited by great gerbils; and (iii) the scale at which plague monitoring is conducted in Kazakhstan. We argue that the network model shows how the three spatial scales are together responsible for the empirical observation of an abrupt threshold for plague.

Percolation theory concerns the behaviour of connected clusters in random networks[4]. Whether an infectious disease will spread among a population of hosts that have a fixed position in space and may only infect their nearest neighbours is well recognized as a percolation problem[14,15]. Equally recognized is the relevance of percolation theory to epidemics on networks in general, and lattice models in particular[16–18]. Among empiricists, the theory has been used to postulate the existence of spatial thresholds for the spread of fungal parasites of plants[2].

A simple case of percolation is when bonds form independently between adjacent points on the plane square lattice with probability $p$. In network terminology, each point on the lattice is a vertex and if a bond is present there is said to be an open edge between the two vertices. A 'cluster' is then defined as a set of vertices connected by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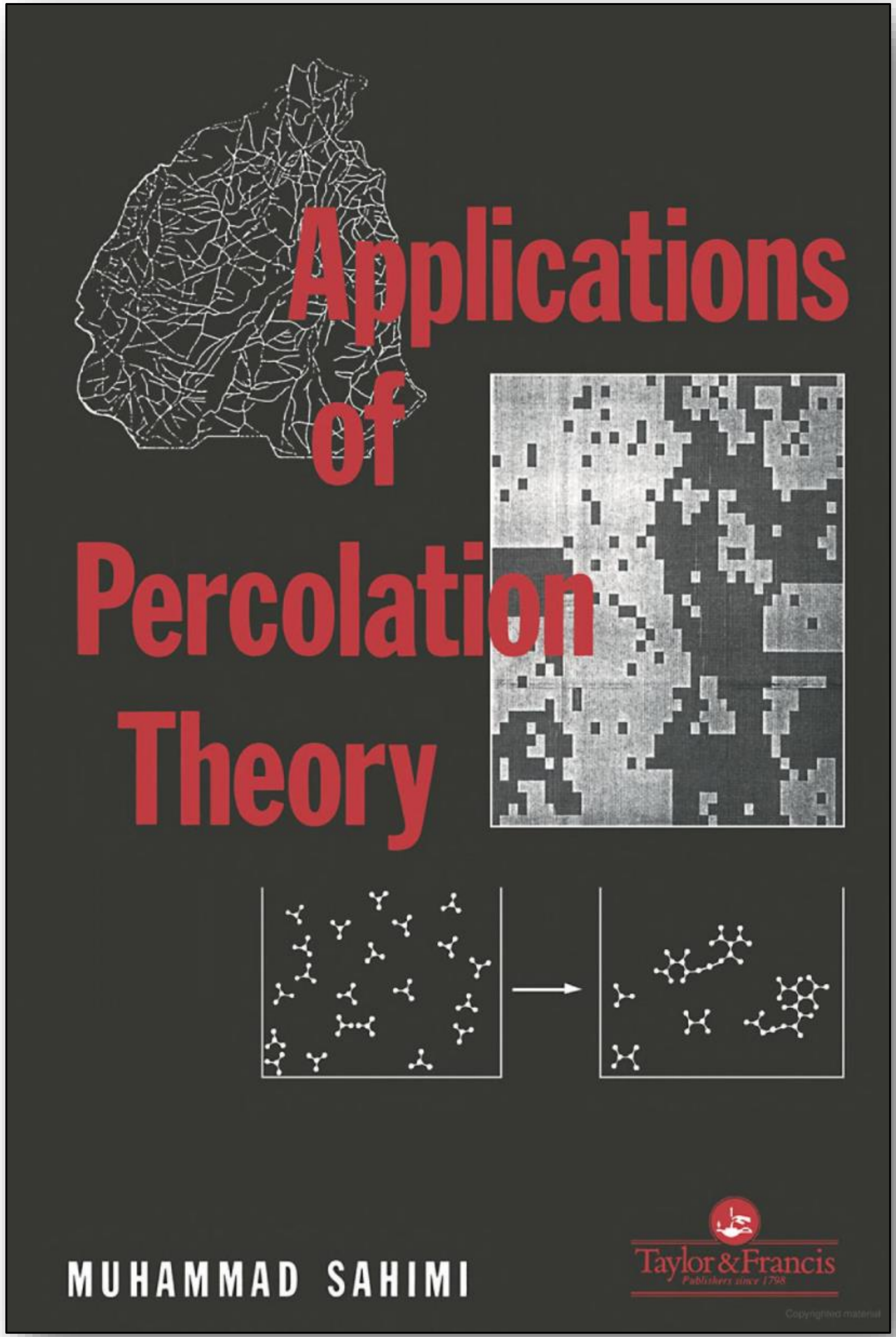
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