MATH 242 — 13 October 2025

The Logistic Map

$$f_r(x) = r \cdot x\,(1-x)$$

parameter: $0 \le r \le 4$

variable: $0 \le x \le 1$

Observations:

If $0 \le r < 1$, then the population converges to 0.

If $1 < r < 3$, then the population converges to a positive value $x^* = \frac{r-1}{r}$.

attracting fixed points (period-1 oscillation)

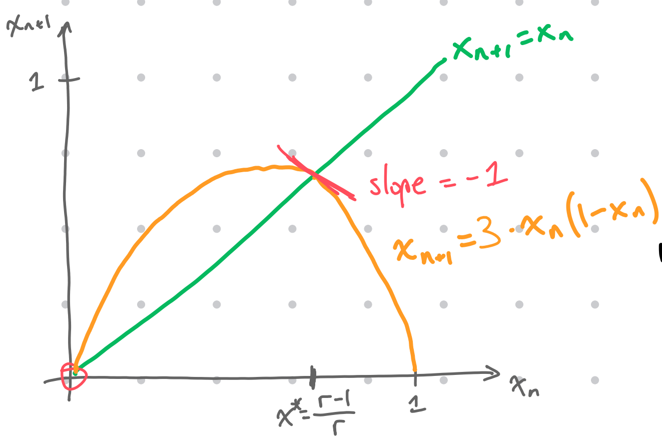
$r=3$ is a bifurcation value

If $3 < r < 3.45$ then the population oscillates between 2 values. — period 2 oscillation

If $3.45 < r <$? then oscillation among 4 values. — period 4 oscillation

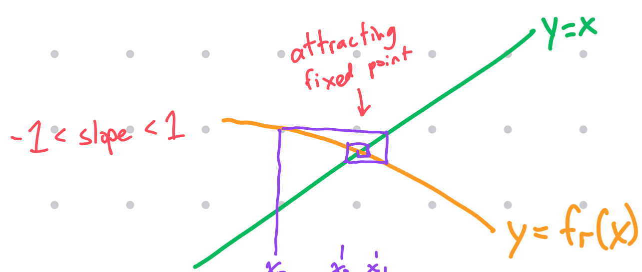
What else happens here?

When r gets close to 4, "crazy" oscillation happens.

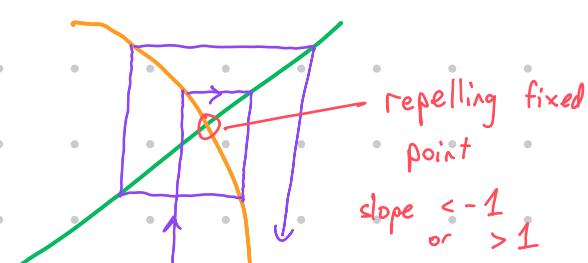


$r \cdot x(1-x)$

Why do some fixed points attract and some repel?



If the slope of $f_r(x)$ at the fixed point $-1 < f_r'(x) < 1$, then the trajectory spirals into the fixed point



If the slope of $f_r(x)$ at the fixed point is $|f_r'(x)| > 1$ then the trajectory spirals away from the fixed point.

If the trajectory oscillates between 2 values:

$a \xrightarrow{f_r} b \rightarrow a \rightarrow b \rightarrow a \rightarrow \cdots$

$b = f_r(a)$

$f_r(b) = f_r(f_r(a)) = a$

$f_r(x) = r \cdot x(1-x)$

Let $f_r^{(2)}(x) = f_r(f_r(x))$.

Then, if f_r oscillates between 2 values, $f_r^{(2)}$ must have a fixed point!

$f_r^{(2)}(x)$ is a 4th-degree polynomial