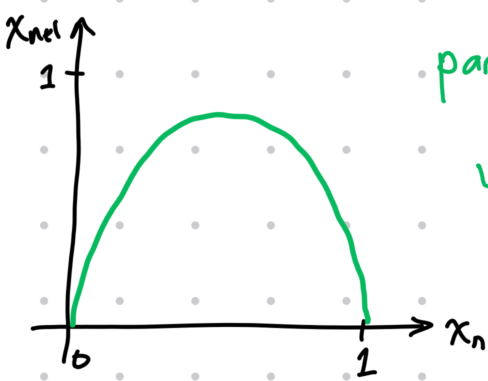
MATH 242 — 10 October 2025

Francis Su writes,
"Even wrong ideas soften the soil in which good ideas can grow"

How have you seen this in your own life or in your own mathematical experience?

SCENARIO: Define a sequence of relative population sizes between 0 and 1: $x_0, x_1, x_2, x_3, \ldots$

Idea: If x_n is close to zero, then so is x_{n+1}.
If x_n is close to 1, then x_{n+1} is close to zero.
If x_n is close to $\frac{1}{2}$, then x_{n+1} is big.



parabola:

$$\underbrace{x_{n+1} = \boxed{r\cdot x_n(1-x_n)}}_{\text{LOGISTIC MAP EQUATION}}$$

define $f_r(x) = r\cdot x(1-x)$

In differential equations, the logistic equation is:

$$\frac{df}{dx} = f(x)(1-f(x))$$

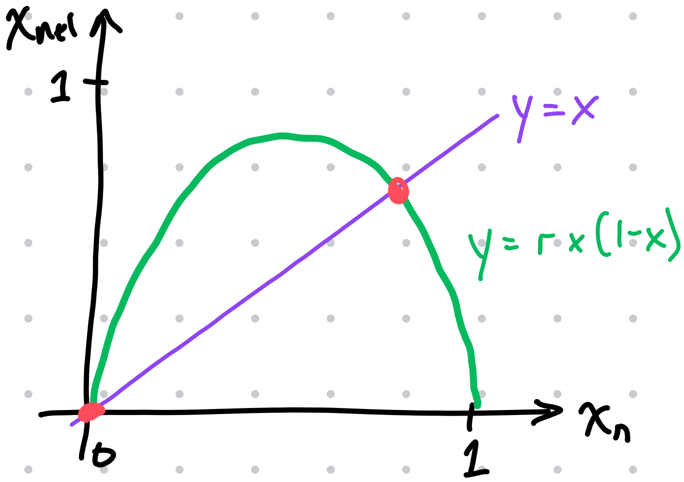
Compute: start with x_0

$x_1 = f_r(x_0)$

$x_2 = f_r(x_1)$

$x_3 = f_r(x_2)$

$\vdots$



Fixed points: a value x^* such that

$$x^* = r\,x^*(1-x^*)$$

solve for x^*:

$\boxed{x^*=0}$ or $1 = r(1-x^*)$

zero fixed point

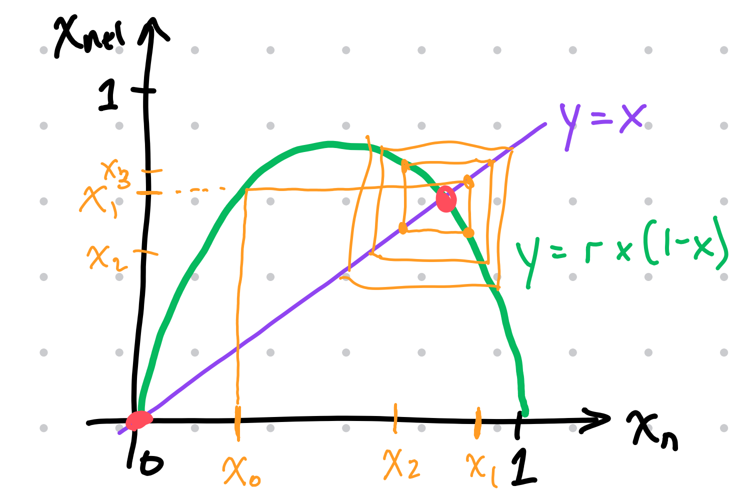
$$\frac{1}{r} = 1 - x^*$$

$$x^* = 1 - \frac{1}{r}$$

nonzero fixed point $\boxed{x^* = \dfrac{r-1}{r}}$

example: $r=2.5$: $x^* = \dfrac{2.5-1}{2.5} = \dfrac{1.5}{2.5} = 0.6$

COBWEB PLOT:



$x_1 = r x_0(1-x_0)$