Math 234

Relations Day 22

1. Let $A = \{n \in \mathbf{Z} \mid -20 \leq n \leq 20\}$ and define relation R by $R = \{(n_1, n_2) \mid n_1^2 = n_2\}$.

   (a) Is it true that $3\ R\ 9$?

   (b) Is it true that $-4\ R\ 16$?

   (c) Is it true that $5\ R\ 10$?

   (d) Write out every element in the set R.

   (e) Write out every element in the set R^{-1}, the inverse relation of R.

   (f) Is the relation R reflexive? Is it symmetric? Is it transitive?

2. For the same set A as above, let $S = \{(n_1, n_2) \mid |n_1| \leq |n_2|\}$.

   (a) Is it true that $-3\ S\ 9$?

   (b) Is it true that $9\ S\ 3$?

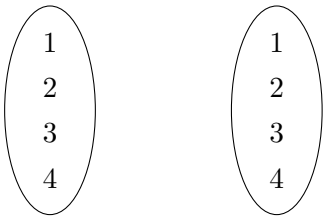
   (c) Is it true that $3\ S\ -9$?

   (d) Is it true that $10\ S^{-1}\ -7$?

   (e) Is the relation S reflexive? Is it symmetric? Is it transitive?

3. Let $A = \{1, 2, 3, 4\}$, and define relation $R = \{(1, 1), (1, 3), (2, 2), (2, 4), (3, 1), (3, 3), (4, 2), (4, 4)\}$.

   (a) Complete the arrow diagram to depict relation R.



   (b) Is R reflexive? Is it symmetric? Is it transitive?

   (c) Draw an arrow diagram to depict relation R^{-1}.

4. Define a relation Q on $\mathbf{R}$ as follows: For all real numbers x and y, $x\ Q\ y \Leftrightarrow x - y$ is rational. Is Q reflexive? Is it symmetric? Is it transitive?

5. Let X be a finite set. Define the following relations on $\mathscr{P}(X)$, the power set of X. Is each relation reflexive? Symmetric? Transitive?

   (a) For all $A, B \in \mathscr{P}(X)$, $A \mathbf{E} B \Leftrightarrow$ the number of elements in A equals the number of elements in B.

   (b) For all $A, B \in \mathscr{P}(X)$, $A \mathbf{L} B \Leftrightarrow$ the number of elements in A is less than the number of elements in B.

   (c) For all $A, B \in \mathscr{P}(X)$, $A \mathbf{N} B \Leftrightarrow$ the number of elements in A is not equal to the number of elements in B.

6. Suppose R and S are reflexive relations on the same set A.

   (a) Is $R \cup S$ a reflexive relation on A? Prove your answer is correct.

   (b) Is $R \cap S$ a reflexive relation on A? Prove your answer is correct.

   (c) Is $R - S$ a reflexive relation on A? Prove your answer is correct.