Math 234

Functions: Inverse and Composition Day 19

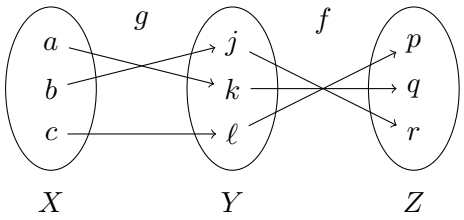
1. Suppose Y and Z are sets and $g : Y \to Z$ is a one-to-one and onto function. What can be inferred in the following situation?

   (a) s_1 and s_2 are elements of Y and $g(s_1) = g(s_2)$.

   (b) $s/2$ and $t/2$ are elements of Y and $g(s/2) = g(t/2)$.

   (c) For some function f whose domain includes x_1 and x_2, $f(x_1)$ and $f(x_2)$ are elements of Y and $g(f(x_1)) = g(f(x_2))$.

2. Let $X = \{a, b, c\}$, $Y = \{d, e, f\}$ and $Z = \{g, h, i\}$. Define $g : X \to Y$ and $f : Y \to Z$ by the diagram below.



   (a) Draw an arrow diagram for the composition $f \circ g$.

   (b) Draw an arrow diagram for the composition $(f \circ g)^{-1}$.

   (c) Draw arrow diagrams for g^{-1} and f^{-1}.

   (d) Draw an arrow diagram for $g^{-1} \circ f^{-1}$.

   (e) How are $(f \circ g)^{-1}$ and $g^{-1} \circ f^{-1}$ related?

3. Suppose $f : X \to Y$ is one-to-one and $g : Y \to Z$ is onto. Consider the composition $g \circ f : X \to Z$.

   (a) Is it *possible* that $g \circ f$ is one-to-one? If so, find examples of functions f and g so that $g \circ f$ is one-to-one. If not, explain why not.

   (b) Is it *certain* that $g \circ f$ is one-to-one? If not, find examples of functions f and g so that $g \circ f$ is not one-to-one. If yes, prove that $g \circ f$ is one-to-one.

   (c) Is it *possible* that $g \circ f$ is onto? If so, find examples of functions f and g so that $g \circ f$ is onto. If not, explain why not.

   (d) Is it *certain* that $g \circ f$ is onto? If not, find examples of functions f and g so that $g \circ f$ is not onto. If yes, prove that $g \circ f$ is onto.

4. Let $f(x) = \dfrac{x+4}{3x-2}$.

(a) The function definition for f is incomplete because the domain of f is not specified. What is the largest set of real numbers that could be the domain of f?

(b) Find the inverse function f^{-1}. What is the domain of f^{-1}?

(c) Check your answer from part (b) by computing $f^{-1}(f(x)) = x$ for all x in the domain of f.

(d) Is f a bijection between two sets of real numbers? If so, which sets? If not, why not?

5. Given any set X and any functions $f : X \to X$, $g : X \to X$, and $h : X \to X$. If h is one-to-one and $h \circ f = h \circ g$, is it true that $f = g$? Either give a proof or a counterexample.

Problem 21 in Section 7.3

6. Given any set X and any functions $f : X \to X$, $g : X \to X$, and $h : X \to X$. If h is one-to-one and $f \circ h = g \circ h$, is it true that $f = g$? Either give a proof or a counterexample.

Problem 22 in Section 7.3