Math 234

Introduction to Probability Day 14

1. For this activity each person needs a standard fair six-sided die.

   (a) Toss the die once and note the number on top. What is the sample space for this experiment?

   $S = \{1, 2, 3, 4, 5, 6\} = \{n \in \mathbb{Z} \mid 1 \le n \le 6\}$

   or $S = \{$⚀, ⚁, ⚂, ⚃, ⚄, ⚅$\}$

   (b) What is the probability that the outcome of 1 occurs from this experiment? *Why?*

   $\frac{1}{6}$

   (c) Now toss the die 24 times, recording the number of 1s, 2s, 3s, 4s, 5s, and 6s that appear. Then combine your counts with the counts from the other students at your table. What proportion of rolls resulted in 1? Does this equal your answer in part (b)? If not, is the discrepancy troubling?

2. Now consider the experiment of tossing your die *twice*, noting the sum of the two numbers that appear on top.

   (a) What is the sample space of this experiment?

   $S = \underbrace{\{(1,1), (1,2), (1,3), \ldots, (6,6)\}}_{36 \text{ pairs (outcomes)}}$

   die A

die B	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

   (b) Are all outcomes equally likely?

   We assume so.

   (c) For this experiment, is getting a sum of 7 in two tosses an outcome or an event?

   event: $A = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\} \subseteq S$

   event: a subset of the sample space

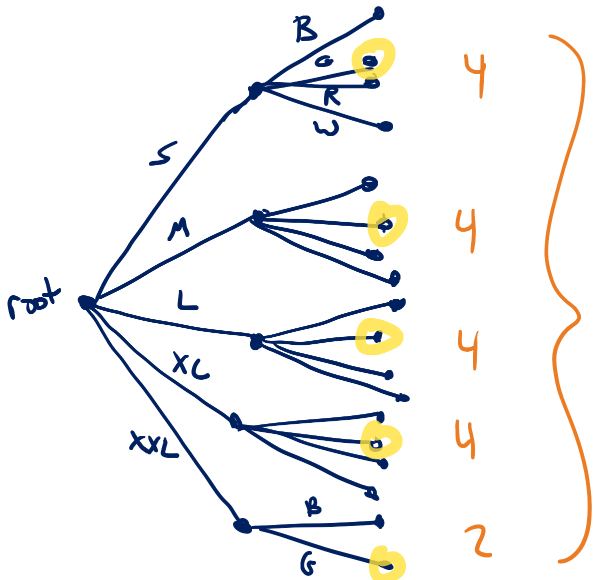
   (d) What is the probability that you get a sum of 7 in two tosses?

   $\frac{6}{36} = \frac{1}{6}$

3. Suppose that St. Olaf T-shirts come in five sizes: S, M, L, XL, and XXL. Further suppose that each size comes in four colors (black, gold, gray, and white) except for XXL, which only comes in black and gold.

B G R W

(a) Draw a tree diagram to indicate all possible size and color pairs. How many different T-shirts does the bookstore need to stock?



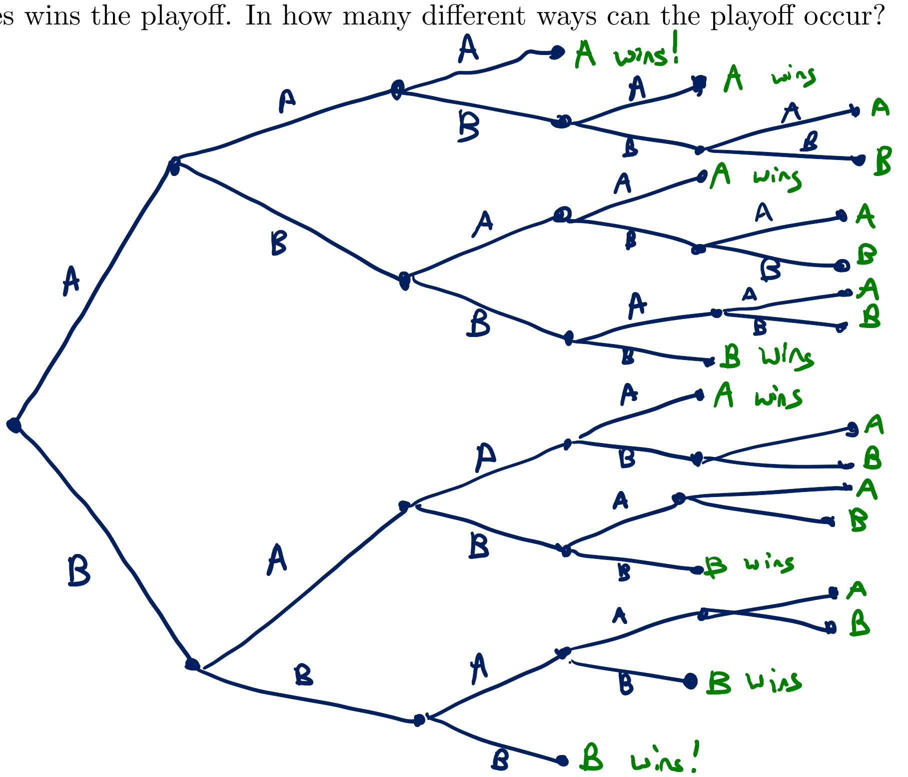
18 different T-shirts

(b) If a T-shirt is selected at random, with each possible size and color pair equally likely, what is the probability that a gold shirt is selected?

$$\frac{5}{18}$$

4. Suppose that a playoff between two teams consists of at most five games. The first team to win three games wins the playoff. In how many different ways can the playoff occur?

Team A
Team B



20 different ways that the playoff can occur

5. Suppose a website enforces the following requirements for passwords: The passwords must be four, five or six characters long. Allowable characters are all uppercase English letters, all lowercase English letters, all digits and the special characters #, %, @, !, and &.

   (a) How many distinct passwords are possible on this system?

   There are 67 allowable characters, so the number of possible passwords is

   $$67^4 + 67^5 + 67^6 = 91,828,658,397$$

   (b) If the website enforces a one second delay between password attempts, how many centuries would a patient robot need to try every valid password?

   $$\frac{91,828,658,397}{60 \cdot 60 \cdot 24 \cdot 365 \cdot 100} = 29.11 \text{ centuries}$$

   (c) Suppose John wants his password to have six characters and to start with the four letters "john" in upper or lower case. How many different passwords are possible for John?

   There are 2 possibilities for each of the first four characters, and 67 possibilities for each of the remaining two characters. The number of possible passwords is thus:

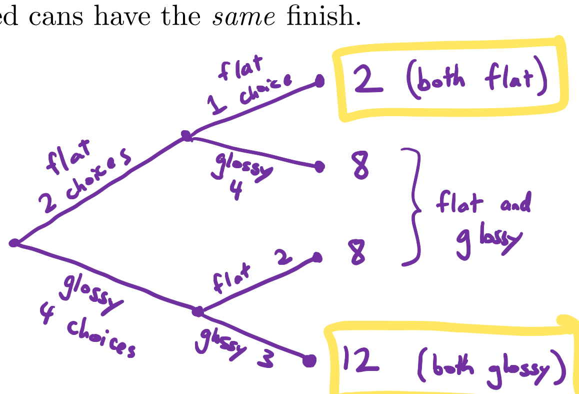
   $$2^4 \cdot 67^2 = 71,824$$

6. A painter has six cans of paint, each containing a different color. Two of the cans contain paint with a flat finish and four of the cans contain glossy paint.

   (a) If the painter selects one can of flat paint and one can of glossy paint, how many different color combinations are possible?

   $$2 \cdot 4 = 8$$

   (b) Suppose the painter forgets that the cans contain paint with different finish, and simply selects two cans at random. Use a tree diagram to help you find the probability that the two selected cans have the *same* finish.



Probability of same finish:

$$\frac{14}{30}$$

7. Minnesota issues license plates that consist of three numbers followed by three letters; for example: 012-ABC. How many different license plates of this form are possible?

10 choices for each of 3 numbers

26 choices for each of 3 letters

Total possible license plates:

$$10^3 \cdot 26^3 = 17{,}576{,}000$$

8. How many different 4-letter codes can be made from the letters in the word *PADLOCKS*, if no letter can be chosen more than once? How about 6-letter codes from the letters in *DOG-WATCHES*?

r-permutations!

Number of ways to arrange 4 of the 8 letters in "PADLOCKS":

$$P(8,4) = \frac{8!}{(8-4)!} = 1680$$

Number of ways to arrange 6 of the 10 letters in "DOGWATCHES":

$$P(10,6) = \frac{10!}{(10-6)!} = 151{,}200$$

9. **Bonus:** A fair coin is flipped repeatedly until heads appears, and the number of flips is recorded.

   (a) What is the probability of each outcome in the sample space $\mathcal{S}$? Show $P(\mathcal{S}) = 1$.

   (b) Let A be the event that an even number of flips are made. What is $P(A)$?

   (c) Let B be the even that at least 3 flips are made. What is $P(B)$?

   (d) What is $P(A \cup B)$?

BONUS

(a) Sample space: $S = \{H, TH, TTH, TTTH, TTTTH, \dots\}$

probabilities: $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{16}$ $\frac{1}{32}$

The probabilities form a geometric sequence whose sum is:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = \frac{\frac{1}{2}}{1 - \frac{1}{2}} = 1$$

(b) $A = \{TH, TTTH, TTTTTH, \dots\}$

Probability that A occurs:

$$P(A) = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots = \frac{\frac{1}{4}}{1 - \frac{1}{4}} = \frac{1}{3}$$

(c) $B = \{TTH, TTTH, TTTTH, \dots\}$

Probability that B occurs:

$$P(B) = \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots = \frac{\frac{1}{8}}{1 - \frac{1}{2}} = \frac{1}{4}$$

(d) $A \cup B = \{TH, TTH, TTTH, TTTTH, \dots\}$

$$P(A \cup B) = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = \frac{\frac{1}{4}}{1 - \frac{1}{2}} = \frac{1}{2}$$