Math 234

Set Theory Day 13

Discuss the following problems with the people at your table.

1. The following is an incomplete proof that for all sets A and B, if $A \subseteq B$, then $A \cup B \subseteq B$. Fill in the blanks to complete the proof.

   Proof: Suppose A and B are any sets and $A \subseteq B$.

   Let $x \in$ _____________.

   By definition of union, $x \in$ _____________ or $x \in$ _____________.

   In the first case, since $A \subseteq B$, $x \in$ _____________.
   In the second case it is clear that $x \in B$.

   Thus, in either case, $x \in$ _____________, which is what we needed to show.

2. Prove that for all sets A and B, $A \cap B \subseteq A$.

3. Consider sets A and B:

$$A = \{m \in \mathbf{Z} \mid m = 5a \text{ for some integer } a\}$$
$$B = \{n \in \mathbf{Z} \mid n = 10b - 15 \text{ for some integer } b\}$$

Prove that $B \subseteq A$.

4. Prove that for all sets A and B, $(B - A) = B \cap A^c$.

5. Prove the following theorem by induction on n, the number of sets in the union.

Theorem: Let $A_1, A_2, A_3, \ldots$ be an infinite collection of sets, and let B be a set. If $A_i \subseteq B$ for all integers $i \geq 1$, then

$$\left(\bigcup_{i=1}^{n} A_i\right) \subseteq B$$

for every positive integer n.

☞ First write the statement $P(n)$ that needs to be proved.

6. Consider the statement: For all sets A and B, if $A \subseteq B$, then $A \cap B^c = \emptyset$.

   Write the negation of the statement. Then prove the statement by contradiction.

7. Prove that for all sets A, B, and C, if $B \cap C \subseteq A$, then $(C - A) \cap (B - A) = \emptyset$.