Math 234

Set Theory Day 12

Discuss the following problems with the people at your table.

1. Consider the following sets: $A = \{1, 3, 6, 10\}$ and $B = \{2, 4, 6, 8\}$. Determine the following sets by writing their elements in set notation:

   (a) $A \cup B = \{1, 2, 3, 4, 6, 8, 10\}$

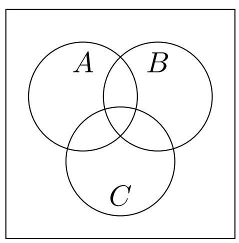
   (b) $A \cap B = \{6\}$

   (c) $B \cap A = \{6\}$

   (d) $A - B = \{1, 3, 10\}$

   (e) $B - A = \{2, 4, 8\}$

2. For each item below, copy the Venn diagram and shade the portion of the Venn diagram corresponding to the indicated set.



   (a) $A \cup B \cup C$

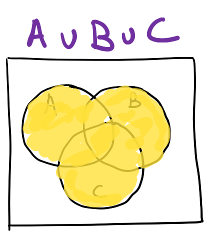
   (b) A^c

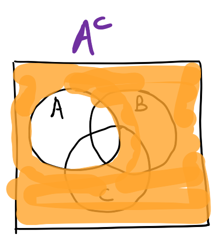
   (c) $A \cup B \cup C^c$

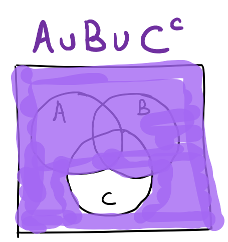
   (d) $(A \cap B) - C$

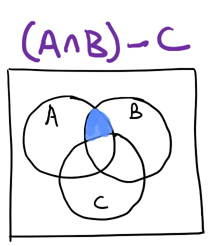
   (e) $A^c \cap B^c \cap C^c$

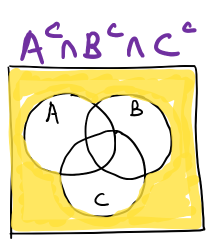
   (f) $(A \cup B \cup C)^c$

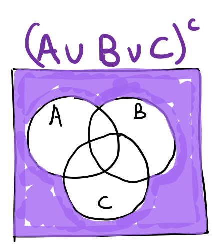












3. Let $A = \{x \in \mathbf{R} \mid i < x < i + 1 \text{ for some integer } i\}$.

(a) Describe in words the set A.

The set of real numbers that are not integers.

(b) Describe in words the set A^c.

$A^c = \mathbb{Z}$

4. Consider the set $A = \{n \in \mathbf{Z} \mid n \text{ is divisible by } 10\}$ and $B = \{n \in \mathbf{Z} \mid n \text{ is divisible by } 20\}$.

$A = \{\ldots, 10, 20, 30, \ldots\}$ $B = \{\ldots, 20, 40, \ldots\}$

(a) Prove that $B \subseteq A$.

Suppose $n \in B$.

That means $n = 20k$ for some $k \in \mathbb{Z}$.

Then $n = 10(2k) = 10m$, where $m = 2k$ is an integer.

Thus, n is a multiple of 10, and so $n \in A$.

So every element of B is also in A, which means $B \subseteq A$.

(b) Prove that $A \not\subseteq B$.

Since $10 \in A$ but $10 \notin B$, $A \not\subseteq B$.

5. Let $C_i = \{-i, i\}$ for all nonnegative integers i.

$C_1 = \{-1, 1\}$

$C_5 = \{-5, 5\}$

$C_0 = \{0\}$

(a) Are C_1 and C_2 disjoint? Are $C_0, C_1, C_2, \dots$ mutually disjoint?

$C_1 = \{-1, 1\}$ $C_2 = \{-2, 2\}$ are disjoint

Yes.

since $C_i \cap C_j = \emptyset$ for any distinct nonnegative integers i and j.

(b) $\bigcup_{i=0}^{4} C_i = ?$

$C_0 \cup C_1 \cup C_2 \cup C_3 \cup C_4 = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

(c) $\bigcap_{i=0}^{4} C_i = ?$ $= \emptyset$

(d) $\bigcup_{i=0}^{n} C_i = ?$ $= \{-n, -n+1, \dots, -3, -2, -1, 0, 1, 2, 3, \dots, n-1, n\}$

(e) $\bigcup_{i=0}^{\infty} C_i = ?$ $= \{n \in \mathbb{Z}\} = \mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

(f) Do the sets $C_0, C_1, C_2, \dots$ form a partition of $\mathbf{Z}$? Yes

6. Let $D = \{1, 4, 7\}$ and $E = \{1, 2\}$.

(a) Write out the Cartesian product $D \times E$.

$D \times E = \{(1,1), (1,2), (4,1), (4,2), (7,1), (7,2)\}$

(b) Write out the power set $\mathscr{P}(D)$.

set of all subsets

$P(D) = \{\emptyset, \{1\}, \{4\}, \{7\}, \{1,4\}, \{4,7\}, \{1,7\}, \{1,4,7\}\}$

(c) How many elements are in $\mathscr{P}(D \times E)$?

$2^6 = 64$

7. If A is a set of n elements, how many elements are in $\mathscr{P}(A)$? Explain your reasoning.

A contains n elements.

Each subset of A either includes or does not include each element of A.

How many subsets?

$$\underbrace{\underline{2}\cdot\underline{2}\cdot\underline{2}\cdot\underline{2}\cdot\underline{2}\ \underline{\cdots}\cdot\underline{2}}_{n} = 2^n$$

8. Given any two sets C and D, describe in words the set $(C \cup D) - (C \cap D)$.

☝ write down some examples for specific sets!

$(C \cup D) - (C \cap D)$ consists of all elements that are in either C or D but not both.

9. **Bonus:** Let $D_i = \left[0, \frac{1}{i}\right] = \left\{x \in \mathbf{R} \mid 0 \leq x \leq \frac{1}{i}\right\}$ for all positive integers i.

(a) What is $\bigcup_{i=1}^{\infty} D_i$?

$$\bigcup_{i=1}^{\infty} D_i = [0, 1]$$

since $D_i \subseteq D_1$ for all $i > 1$.

(b) What is $\bigcap_{i=1}^{\infty} D_i$?

$\bigcap_{i=1}^{\infty} D_i = \{0\}$ since 0 is the only number in <u>all</u> of the sets D_i.