Math 234

Sequences Day 9

Discuss the following problems with the people at your table.

1. A sequence is defined by $a_k = \frac{k}{2k+6}$. Write the terms a_1, a_2, a_3, and a_4.

2. Find an explicit formula for each sequence $a_1, a_2, a_3, \ldots$ below.

   (a) $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \ldots$

   (b) $\frac{2}{2}, \frac{4}{3}, \frac{6}{4}, \frac{8}{5}, \frac{10}{6}, \ldots$

   (c) $1, \frac{4}{3}, \frac{3}{2}, \frac{8}{5}, \frac{5}{3}, \ldots$

   (d) $\frac{-1}{3}, \frac{1}{6}, \frac{-1}{11}, \frac{1}{18}, \frac{-1}{27}, \ldots$

   (e) $\frac{1}{3}, \frac{-2}{7}, \frac{3}{13}, \frac{-4}{21}, \frac{5}{31}, \ldots$

3. Compute the sum $\sum_{m=0}^{3} \frac{1}{2^m}$.

4. Compute the product $\prod_{k=1}^{3} \left(1 + \frac{1}{k}\right)$

5. Write the following in summation notation:

$$(1^5 - 1) + (2^5 - 1) + (3^5 - 1) + (4^5 - 1) + (5^5 - 1)$$

6. Write the following in product notation:

$$\left(\frac{2}{4}\right)\left(\frac{3}{5}\right)^2\left(\frac{4}{6}\right)^3\left(\frac{5}{7}\right)^4$$

7. Transform the following by making the change of variables $j = i - 1$

$$\sum_{i=1}^{n-1} \frac{1}{(n-i)^2}$$

8. Transform the following by making the change of variables $k = i + 1$

$$\sum_{i=0}^{n} \frac{i}{i^2 + 1}$$

9. Simplify the expressions:

   (a) $\dfrac{100!}{98!}$

   (b) $\dfrac{n!}{(n-3)!}$

   (c) $\dfrac{n!}{(n-k)!}$

10. Compute the value of the combinations:

    (a) $\binom{10}{8}$

    (b) $\binom{n}{n-2}$ assume $n \geq 2$

11. **Bonus:** problem 66 in section 5.1 of the textbook

    (a) Prove that $n! + 2$ is even for all integers $n \geq 2$.

    (b) Prove that $n! + k$ is divisible by k for all integers $n \geq 2$ and $k \in \{2, 3, \ldots, n\}$.

    (c) Given any integer $m \geq 2$, does there exist a sequence of $m - 1$ consecutive positive integers none of which is prime? Explain.