Math 234

Cases, Contradiction, and Contraposition — Day 8

Discuss the following problems with the people at your table.

1. Warm-up problems about quotients and remainders:

   (a) Compute the values of 39 div 15 and 39 mod 15.

   (b) Compute the values of -27 div 11 and -27 mod 11.

   (c) Today is a Monday. What day of the week will it be:

   Two days from now?

   100 days from now?

   400 days from now?

2. Prove that for all integers n, $n^2 - n + 3$ is odd.

☞ Break this problem into cases.

3. Prove that for any integer n, the quantity $(n^3 - n)(n + 2)$ is divisible by 4. cases?

4. Consider the statement: *The cube root of any irrational number is irrational.*

   (a) Write the negation of the statement.

   (b) Show that assuming the negation is true leads to a contradiction.

   (c) What does this imply about the original statement?

5. Let a, b and c be prime numbers greater than two. Use contradiction to prove that $a + b^2 \neq c^2$.

☞ What is the statement we want to prove? What is its negation?

6. Consider the statement: *The negative of any irrational number is irrational.*

   (a) Write this statement in the form "$\forall x$ in D, if $P(x)$ then $Q(x)$." Then write the contrapositive of this statement.

   (b) Prove the contrapositive.

   (c) What does your proof imply about the original statement?

7. Using contraposition, prove the following statement: *For all integers m and n, if $m + n$ is even, then either both m and n are even or both m and n are odd.*

8. Consider the statement: *The sum of any two irrational numbers is irrational.*

   (a) What is wrong with the following "proof" of this statement?

   Incorrect proof: Let a and b be any two irrational numbers. The number a cannot be written as the quotient of two integers, nor can b. Therefore, the sum $a+b$ cannot be written as the quotient of two integers, so $a + b$ must be irrational.

   (b) Is the statement true or false? Provide a proof or a counterexample.