Math 234

Direct Proof and Counterexample — Day 6

Discuss the following problems with the people at your table.

1. Assume that m and n are integers.

   (a) Prove that $14m + 6n + 5$ is odd.

   (b) Prove that $14m + 6n - 10$ is even.

2. Show by a counterexample that the following statement is false: "For any two prime numbers m and n, the sum $m + n$ is a composite number."

3. In this problem you may use the facts that $(-1)^2 = 1$ and $1^k = 1$ for any integer k. Write a formal proof of each statement below:

   (a) If n is an even integer, then $(-1)^n = 1$.

   (b) If n is an odd integer, then $(-1)^n = -1$.

4. Prove or disprove the statement: "If k is an odd integer and m is an even integer, then $k^2 + m^2$ is odd."

5. Is $0.42424242\ldots$ a rational number? Why or why not?

6. Is $0.123123123\ldots$ a rational number? Why or why not?

7. Prove the statement: "If k is a rational number and m is a rational number, then $k^2 + m^2$ is a rational number." You may use the fact that if n and j are integers, so is the quantity n^j.

8. Let r and s be arbitrary rational numbers. Decide whether each of the following statements is true or false and provide a proof of your assertion.

   (a) $3r + 2s$ is rational.

   (b) $19r - 4s + \frac{r}{s}$ is rational.

9. Suppose a, b, c and d are integers. Also suppose x is a real number that satisfies the equation

$$\frac{ax+b}{cx+d} = 1.$$

(a) If the condition that $a \neq c$ is added, decide whether x must be rational and prove the correctness of your assertion.

(b) If we know $a = c$, must x be rational? Prove your answer is correct.

(c) Define the following predicates:

$P(a, b, c, d, x)$ is "x solves the equation $\frac{ax+b}{cx+d} = 1$"
$Q(a, c)$ is "$a = c$"
$R(x)$ is "x is rational"

Use formal logic notation to express the statement "If $a = c$ and x solves the equation, then x must be rational." What is the negation of this statement? (In this problem you can assume a, b, c and d are understood to be integers. You needn't express this explicitly.)