Math 234

Logical Statements — Day 2

Discuss the following problems with the people at your table.

1. First, replace statements with letters to write the logical form of Argument 1.

   Argument 1:

   $\underbrace{\text{This day is sunny}}_{p}$ or $\underbrace{\text{this day is cloudy}}_{q}$.
   This day is not cloudy.
   Therefore, this day is sunny.

   $p \vee q$
   $\sim q$
   $\therefore p$

   Given that Argument 2 has the same logical form as Argument 1, what statement goes in the blank?

   Argument 2:

   $\underbrace{\text{This sport is in the Olympics}}_{p}$ or $\underbrace{\text{this sport is not serious}}_{q}$.
   $\sim q$: *This sport is serious.*
   Therefore, this sport is in the olympics.

definition of $\vee$:

p	q	$p \vee q$	$p \oplus q$
T	T	T	F
T	F	T	T
F	T	T	T
F	F	F	F

2. Define the following statements:

$$m = \text{"This house has green paint on its exterior."}$$
$$n = \text{"This house has gold paint on its exterior."}$$
$$p = \text{"This house is in Wisconsin."}$$

Now write the following statements in symbolic form using the symbols $\vee$, $\wedge$, and $\sim$.

(a) This house has both green and gold paint, but is not in Wisconsin.

$(m \wedge n) \wedge \sim p$

(b) This house is in Wisconsin and has green or gold paint. (or both)

$p \wedge (m \vee n) \equiv p \wedge ((m \vee n) \vee (m \wedge n))$

(c) This house is in Wisconsin and has green or gold paint, but not both. Be careful!

$p \wedge (m \vee n) \wedge \sim(m \wedge n) \equiv p \wedge (m \oplus n)$

exclusive or

(d) This house has neither green nor gold paint.

$\sim m \wedge \sim n \equiv \sim(m \vee n)$ ← De Morgan's Laws

(e) This house has neither green nor gold paint but is in Wisconsin.

$p \wedge \sim(m \vee n) \equiv (\sim m \wedge \sim n) \wedge p$

3. For each of the following statements, determine whether an *inclusive or* or an *exclusive or* is intended. Explain your answers.

   (a) Coffee or tea comes with dinner.

   exclusive — in some cases, inclusive

   (b) A password must contain letters and numbers or be at least 8 characters long.

   inclusive

   (c) The prerequisite for this course is CSCI 221 or MATH 126.

   inclusive

   (d) Publish or perish.

   exclusive

4. Complete the following truth table for the statement $(p \vee q) \vee (\sim p \wedge \sim q)$.

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim p \wedge \sim q$	$(p \vee q) \vee (\sim p \wedge \sim q)$
T	T	F	F	T	F	T
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	T

What kind of statement is $(p \vee q) \vee (\sim p \wedge \sim q)$?

tautology

5. For each pair of statements below, create two truth tables to determine whether the statements are logically equivalent.

   (a) $\sim(p \vee q)$ ~~and~~ $\equiv$ $\sim p \wedge \sim q$

p	q	$\sim(p \vee q)$ $\equiv$	$\sim p \wedge \sim q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

   de Morgan's Laws

   (b) $\sim(p \wedge q)$ ~~and~~ $\equiv$ $\sim p \vee \sim q$

p	q	$\sim(p \wedge q)$ $\equiv$	$\sim p \vee \sim q$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

6. Come up with a statement form involving p and q that is a contradiction.

7. Express the following statement using logical symbols:

$\underbrace{\text{The automated reply cannot be sent}}_{p}$ when $\underbrace{\text{the file system is full}}_{q}$.

If q, then p.

$q \rightarrow p$ q "implies" p

q	p	$q \rightarrow p$
T	T	T
T	F	F
F	T	T
F	F	T

$q \rightarrow p$ is vacuously true

8. Complete the following truth table to compare the truth values of the statements $p \rightarrow q$, $q \vee \sim p$, and $p \wedge \sim q$.

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$q \vee \sim p$	$p \wedge \sim q$
T	T	F	F	T	T	F
T	F	F	T	F	F	T
F	T	T	F	T	T	F
F	F	T	T	T	T	F

$$(p \rightarrow q) \equiv (q \vee \sim p)$$

What statement is logically equivalent to $\sim(p \rightarrow q)$?

This is the *negation* of $p \rightarrow q$.

$$\sim(p \rightarrow q) \equiv (p \wedge \sim q)$$

9. Write the negation, converse, inverse, and contrapositive of the following statement.

If $\underbrace{\text{you have a ticket}}_{p}$, then $\underbrace{\text{you can board the flight}}_{q}$. $\rightarrow$

(a) Negation: It is not the case that ________________.

$p \wedge \sim q$ You have a ticket and cannot board the flight.

(b) Converse:

$q \rightarrow p$ If you can board the flight, then you have a ticket.

(c) Inverse:

$\sim p \rightarrow \sim q$ If you don't have a ticket, then you cannot board the flight.

(d) Contrapositive:

$\sim q \rightarrow \sim p$ If you cannot board the flight, then you don't have a ticket.

10. Write the negation, converse, inverse, and contrapositive of the following statement.

T *If my major is computer science, then I take Math 234.*

(a) Negation:

(b) Converse:

F If I take 234, then my major is CS.

(c) Inverse:

F If my major is not CS, then I don't take 234.

(d) Contrapositive:

T If I don't take 234, then my major is not CS.

— class ended here —

11. Which of the following are logically equivalent?

a statement | its converse | its inverse | its contrapositive

12. Determine whether $(p \to r) \vee (q \to r)$ and $(p \wedge q) \to r$ are logically equivalent.

p	q	r	$p \to r$	$q \to r$	$(p\to r) \vee (q \to r)$	$p \wedge q$	$(p \wedge q) \to r$
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	F	T	T	T	T	F	T
T	F	F	F	T	T	F	T
F	T	T	T	T	T	F	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

These columns are the same, so:

$$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$$