Final Exam: Take-Home

Name: _____

Math 230: Differential Equations

Due at the final exam session (Thursday, Nov. 19 at 9:00am)

Take-Home Instructions:

1. For this part of the exam, you may use your textbook, your notes, the course web site, and computing technology (e.g. *Mathematica*).
2. Do not consult other sources, people, web sites, etc. If you have questions about the exam, email the professor.
3. Answer the following questions in the space provided. You may attach extra papers if necessary. Check your work.
4. The St. Olaf Honor Code applies to this exam. Remember the pledge on page 4.

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1. (12 points) Consider the following system:

$$\frac{dx}{dt} = 4y(x-2) - 2x$$
$$\frac{dy}{dt} = 3x^2 - 3y$$

(a) Find equations of the nullclines of this system.

(b) Find and classify all equilibrium points of this system.

(c) Sketch a graph that clearly shows the nullclines and all equilibrium points.

(d) Either find a Hamiltonian function for this system, or show that the system is not Hamiltonian.

(e) What is the long-term behavior of the solution with initial condition $(x, y) = (-1, -1)$?

2. (8 points) Consider the initial-value problem

$$\frac{dy}{dt} = 3y - 4t - 1, \qquad y(0) = 1.$$

Use the Runge-Kutta method to approximate $y(3)$ with 10, 20, 40, and 80 steps. Then find the exact solution $y(t)$, and compute the error for each of your approximations. Write your results in the following table.

number of steps	**step size**	**approximation of $y(3)$**	**error**
10			
20			
40			
80			

What is the exact solution $y(t)$ to the initial-value problem?

Please attach computational output that justifies at least one of the approximations that you reported in the table above.

3. (3 points) What was one thing you found difficult in this course? How have you been able to overcome this difficulty? Has this experience helped you develop a growth mindset toward learning and doing mathematics? How so? Explain.

(There are no wrong answers, but you must explain your answer.)

St. Olaf Honor Pledge: I pledge my honor that on this examination I have neither given nor received assistance not explicitly approved by the professor and that I have seen no dishonest work.

Signed: _____

☐ I have intentionally not signed the pledge. (Check the box if appropriate.)