Practice Problems

Math 230

1. Consider the system of nonlinear differential equations:

$$\frac{dx}{dt} = 10 - x^2 - y^2$$
$$\frac{dy}{dt} = 3x - y$$

(a) Find and classify all equilibrium points.

(b) Sketch the nullclines. Then sketch the phase portrait.

2. Consider the differential equation $\frac{dy}{dt} = f(t)$ with $y(a) = 0$.

(a) Explain why approximating $y(b)$ using Euler's method is the same as approximating the integral $\int_a^b f(t)\ dt$ by a Riemann sum using the values at left endpoints of subintervals.

(b) Explain why approximating $y(b)$ using *improved* Euler's method is the same as approximating $\int_a^b f(t)\ dt$ using the Trapezoid Rule.

3. Exploration: Implement a "midpoint method" for approximating the solution to $\frac{dy}{dt} = f(t, y)$ with $y(0) = y_0$. That is, let y_{k+1} be determined from y_k by:

$$y_{k+1} = y_k + f\left(t_k + \frac{\Delta t}{2},\ y_k + \frac{\Delta t}{2} f(t_k, y_k)\right) \Delta t.$$

Test this method for various choices of $f(t, y)$. How does this method compare with improved Euler's method? Which has smaller error?