Cauchy-Euler Equations
Math 230

1. Consider the differential equation $t^2 \dfrac{d^2 y}{dt^2} - y = 0$.

    (a) Find the power series solution.

    (b) Do you think you have found all solutions to the differential equation? Why or why not?

    (c) Look for a solution of the form $y(t) = t^r$. For what values of r is this a solution?

    (d) Does this differential equation have a unique solution through every point (t_0, y_0)?

2. Using your knowledge from Problem 1, find two linearly independent, non-trivial solutions to

$$t^2\frac{d^2y}{dt^2} + 5t\frac{dy}{dt} - 5y = 0.$$

3. Find two linearly independent, non-trivial, real-valued solutions to $t^2\frac{d^2y}{dt^2} - t\frac{dy}{dt} + 5y = 0$.

Note that complex exponents can be handled as follows:

$$t^{ib} = e^{\ln(t^{ib})} = e^{ib\ln(t)} = \cos(b\ln t) + i\sin(b\ln t)$$

Can you justify each equal sign above?

4. Generalize: what are the solutions to $t^2\frac{d^2y}{dt^2} + at\frac{dy}{dt} + by = 0$ for real constants a and b?