Power Series Solutions to Differential Equations

Math 230

1. Consider the differential equation $\dfrac{dy}{dt} = 3y$.

(a) Find the power series solution, up to and including the t^4 term.

(b) Use the initial condition $y(0) = 2$ to solve for the coefficient in your power series solution.

(c) What is the exact solution to the differential equation? Check that the Taylor series for this solution agrees with your power series, up to the t^4 term.

Taylor Series Reference

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

2. Find the power series solution to $\frac{d^2y}{dt^2} + t\frac{dy}{dt} + y = 1$, up to and including the t^5 term. Make sure only two unknown constants appear in your solution.

3. Find the power series solution to $\frac{dy}{dt} = y + \sin(2t)$, up to and including the t^5 term. Only one unknown constant should appear in your solution.

4. Find the power series solution to $\frac{dy}{dt} = y^2$. Your solution should contain only one unknown constant. What simple function has a power series that matches what you found?