Nonlinear Systems – Nullclines and Bifurcations

Math 230

Working in groups, we will describe the long-term behavior of solutions to following competing species model:

$$\begin{aligned}\frac{dx}{dt} &= x(1 - x - ay)\\ \frac{dy}{dt} &= 2y\left(1 - \frac{x}{2} - y\right)\end{aligned}$$

In this model, a is a positive parameter, and we only consider where $x, y \geq 0$.

1. First, let $a = \frac{1}{2}$.

Half of your group should find and classify the equilibrium points. (*Hint:* linearization!)

The other half should compute the nullclines and the direction of the solutions along them.

Then combine your information to sketch the phase portrait and describe possible solution behaviors.

2. Now, let $a = 2$. Switching roles from before, classify equilibrium points or sketch the nullclines and direction arrows. Then combine this information, sketch the phase portrait, and describe possible solutions behaviors.

3. At what value of a does a bifurcation occur? Recall that a bifurcation occurs when there is a qualitative change in the solutions to the system.

Qualitative Analysis of Nonlinear Systems

Math 230

Do a qualitative analysis of the system:

$$\frac{dx}{dt} = \sin(\pi x)$$
$$\frac{dy}{dt} = x - y^2$$

That is, paint as complete a picture as you can of the phase portrait and solution behaviors.