Trace-Determinant Plane
Math 230

1. Consider the one-parameter family of linear systems $\dfrac{d\mathbf{Y}}{dt} = \mathbf{AY}$, where $\mathbf{A} = \begin{bmatrix} 0 & a \\ 2 & 1 \end{bmatrix}$ depends on the parameter a.

(a) As a varies from a large negative number to a large positive number, the corresponding point (T, D) in the trace-determinant plane moves along what curve? Draw a sketch.

(b) At what value(s) of a does a bifurcation occur? That is, at what value(s) of a does the type of phase portrait change?

(c) Describe what happens to the straight-line solutions as a approaches $-\frac{1}{8}$ from below. Sketch phase portraits for solutions corresponding to a slightly smaller than, equal to, and slightly greater than $-\frac{1}{8}$.

(d) Describe what happens to the solutions as a approaches 0 from below. Sketch phase portraits for solutions corresponding to a slightly smaller than, equal to, and slightly greater than 0.

2. Now let $\mathbf{B} = \begin{bmatrix} b & 1-b \\ 1 & b \end{bmatrix}$, and consider the one-parameter family of linear systems $\frac{d\mathbf{Y}}{dt} = \mathbf{BY}$. Sketch the corresponding curve in the trace-determinant plane. Identify and describe all bifurcations.