Linear Systems with Repeated or Zero Eigenvalues

Math 230

Consider the matrix $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$.

1. Find the eigenvalues and eigenvectors of $\mathbf{A}$.

2. Find the solution with each of the following initial conditions:

(a) $\mathbf{Y}(0) = (1, 1)$

(b) $\mathbf{Y}(0) = (1, 2)$

(c) $\mathbf{Y}(0) = (x_0, y_0)$

3. What do you think the phase portrait looks like for this system?

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4. Suppose a 2×2 matrix $\mathbf{B}$ has only one eigenvalue but *two* linearly independent eigenvectors. What can you say about matrix $\mathbf{B}$? (*Can you come up with any matrices with this property?*)

5. For any matrix $\mathbf{B}$ that you found above, what is the general solution to $\frac{d\mathbf{Y}}{dt} = \mathbf{BY}$?

Consider the matrix $\mathbf{C} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

6. Find the eigenvalues and eigenvectors of $\mathbf{C}$.

7. Find the solution with each of the following initial conditions:

(a) $\mathbf{Y}(0) = (2, -2)$

(b) $\mathbf{Y}(0) = (3, -1)$

(c) $\mathbf{Y}(0) = (x_0, y_0)$

8. What do you think the phase portrait looks like for this system?

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9. Suppose a 2×2 matrix $\mathbf{D}$ has only one eigenvalue, which is zero. What can you say about matrix $\mathbf{D}$? (*Can you come up with any matrices with this property?*)

10. For any matrix $\mathbf{D}$ that you found above, what is the general solution to $\frac{d\mathbf{Y}}{dt} = \mathbf{DY}$?