## Linear Systems with Complex Eigenvalues

Math 230

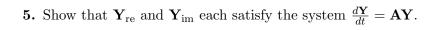
Consider the matrix  $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ .

1. Find the eigenvalues of A. Note that they are complex conjugates.

2. Let  $\lambda$  be the eigenvalue with positive imaginary part. Find an eigenvector V associated with  $\lambda$ .

**3.** Write down the complex-valued "straight-line" solution  $\mathbf{Y}_1 = \mathbf{V}e^{\lambda t}$  to the system  $\frac{d\mathbf{Y}}{dt} = \mathbf{A}\mathbf{Y}$ .

**4.** Use Euler's formula,  $e^{i\theta} = \cos \theta + i \sin \theta$ , to expand this solution. Then collect the real and imaginary parts so that  $\mathbf{Y}_1 = \mathbf{Y}_{\text{re}} + i \mathbf{Y}_{\text{im}}$ , where  $\mathbf{Y}_{\text{re}}$  and  $\mathbf{Y}_{\text{im}}$  are real-valued functions.



**6.** Write down the general solution to  $\frac{d\mathbf{Y}}{dt} = \mathbf{AY}$ .

7. Find a solution with the initial value  $\mathbf{Y}(0) = (2, 4)$ .

8. What is the long-term behavior of the solution you found?