Linear Systems with Real Eigenvalues

Math 230

Consider the linear system:

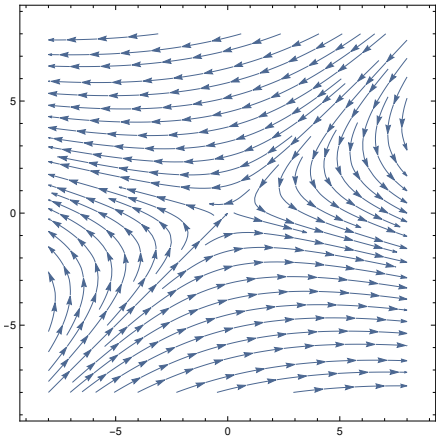
$$\frac{dx}{dt} = 2x - 4y$$
$$\frac{dy}{dt} = -x - y$$

1. Compute the eigenvalues and eigenvectors of the coefficient matrix for this system.

2. Find two straight-line solutions and the general solution for this system.

3. Compute the solution to the system with initial value $x(0) = 6$, $y(0) = 1$.

4. On the plot below, draw the straight-line solutions and the solution to the initial-value problem.



Plot produced in Mathematica by: `StreamPlot[{2x - 4y, -x - y}, {x, -8, 8}, {y, -8, 8}]`

5. Why do the solution trajectories seem to asymptotically approach one of the straight-line solutions, and not the other one?

Extra practice: For each of the matrices below,

(a) Find the eigenvalues and corresponding eigenvectors.

(b) Match the phase plane with one of the diagrams at the bottom of this page.

(c) Describe the long-term behavior of solutions to the system.

1. $\mathbf{A} = \begin{bmatrix} 5 & -2 \\ -1 & 4 \end{bmatrix}$

2. $\mathbf{A} = \begin{bmatrix} -3 & 6 \\ 3 & 0 \end{bmatrix}$

3. $\mathbf{A} = \begin{bmatrix} -4 & -2 \\ -1 & -5 \end{bmatrix}$