Linear Systems

Math 230

1. If $\mathbf{Y} = \begin{bmatrix} r \\ s \end{bmatrix}$, is a nonzero equilibrium solution of $\frac{d\mathbf{Y}}{dt} = \mathbf{A}\mathbf{Y}$, what does this imply about $\det(\mathbf{A})$?

2. For which of the following matrices $\mathbf{A}$ does the system $\frac{d\mathbf{Y}}{dt} = \mathbf{A}\mathbf{Y}$ have nontrivial equilibrium solutions?

$$\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix} \qquad \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$$

3. Suppose that $\mathbf{Y} = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ is a solution of $\frac{d\mathbf{Y}}{dt} = \mathbf{A}\mathbf{Y}$. Show that $k\mathbf{Y}$ is also a solution for every constant k.

4. Suppose that $\mathbf{Y}_1$ and $\mathbf{Y}_2$ are solutions of $\frac{d\mathbf{Y}}{dt} = \mathbf{A}\mathbf{Y}$. Show that $\mathbf{Y}_1 + \mathbf{Y}_2$ is also a solution.

5. Let $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$, $\mathbf{Y}_1 = \begin{bmatrix} e^{-t} \\ -2e^{-t} \end{bmatrix}$, and $\mathbf{Y}_2 = \begin{bmatrix} e^{3t} \\ 2e^{3t} \end{bmatrix}$.

(a) Verify that $\mathbf{Y}_1$ and $\mathbf{Y}_2$ satisfy $\frac{d\mathbf{Y}}{dt} = \mathbf{AY}$.

(b) Find a solution to $\frac{d\mathbf{Y}}{dt} = \mathbf{AY}$ with initial condition $\mathbf{Y}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.