Predator-Prey Systems
## Math 230

1. Consider the system of differential equations modeling two competing species, x and y:

$$\frac{dx}{dt} = 4x - 2xy$$
$$\frac{dy}{dt} = -3y + 3xy$$

(a) Which species (predators or prey) is represented by x, and which is represented by y? How do you know?

(b) Find all equilibrium solutions of this system.

(c) From the system of differential equations, what can you say about the short-term behavior of the solution initial conditions $x(0) = 3$ and $y(0) = 1$?

(d) The following is a phase portrait for this predator-prey system. What does this plot tell you about the long-term behavior of the solution with initial conditions $x(0) = 3$ and $y(0) = 1$? Sketch the solution curve on this plot.
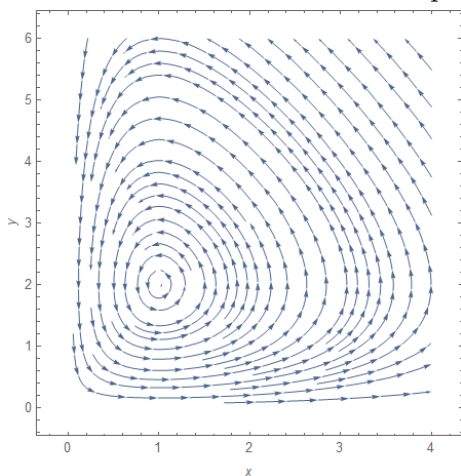
(e) Sketch the $x(t)$ and $y(t)$ graphs for the solution with initial conditions $x(0) = 3$ and $y(0) = 1$.

2. Consider the system of differential equations modeling two species, x and y:

$$\frac{dx}{dt} = 2x\left(1 - \frac{x}{3}\right) - xy$$
$$\frac{dy}{dt} = 3y\left(1 - \frac{y}{5}\right) - 3xy$$

(a) How would you describe the relationship between the members of these species? Are they predators and prey? Interdependent? Competing? *Justify your answers.*

(b) Find all equilibrium solutions of the system.

(c) What can you say about the short-term behavior of the solution to the system with initial conditions $x(0) = 4$ and $y(0) = 0$? What can you say about the long-term behavior?

(d) What can you say about the short-term behavior of the solution to the system with initial conditions $x(0) = 1$ and $y(0) = 1$? What can you say about the long-term behavior?

Hint: You may wish to consider the phase portrait. To draw the phase portrait in Mathematica, use the following command:

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StreamPlot[{2x(1 - x/3) - x*y, 3y(1 - y/5) - 3x*y}, {x,0,4}, {y,0,6}, FrameLabel->{x, y}]
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(e) What long-term behaviors are possible for solutions with other initial conditions?