One-Parameter Families and Bifurcations

Math 230

In problems 1–4, sketch the phase line for each differential equation. Identify equilibrium points as souces, sinks, or nodes. Then sketch the family of solution curves $y(t)$.

1. Consider the one-parameter family $\frac{dy}{dt} = y^4 + ky^2$.

(a) Find all equilibrium solutions. How do they depend on k? (Consider all real numbers k.) *Check your answers with your group members.*

(b) Draw phase lines for selected values of k. Be sure to include at least one value of k in each case you found in part (a). Also sketch some solutions for each case.

(c) For what values of k do bifurcations occur? Discuss with your group members how the equilibrium points change as k increases through each bifurcation value.

2. Consider the one-parameter family $\frac{dy}{dt} = (y^2 - \alpha - 1)(y^2 - 1)$.

(a) Find all equilibrium solutions. How do they depend on k? (Consider all real numbers k.) *Check your answers with your group members.*

(b) Draw phase lines for selected values of k. Be sure to include at least one value of k in each case you found in part (a). Also sketch some solutions for each case.

(c) For what values of k do bifurcations occur? Discuss with your group members how the equilibrium points change as k increases through each bifurcation value.

3. Sketch a function $f(y)$ such that the 1-parameter family of differential equations $\frac{dy}{dt} = f(y) + \alpha$ satisfies the following:

- If $\alpha \leq -3$, there is exactly 1 equilibrium point, a source.
- If $\alpha \geq 2$, there is exactly 1 equilibrium point, a source.
- At $\alpha = 0$, there are exactly 4 equilibrium points.