Existence and Uniqueness

Math 230

1. Suppose $f(t, y)$ and $\frac{\partial y}{\partial t}$ are continuous for all (t, y). Also suppose that $y_1(t) = 3$, $y_2(t) = 6$, and $y_3(t) = t^2 + 8$ are solutions to $\frac{dy}{dt} = f(t, y)$ for all t.

   (a) If a particular solution satisfies $y(0) = 4$, explain why $3 < y(t) < 6$ for all t for this solution.

   (b) What lower and upper bounds can you give for a particular solution that satisfies $y(0) = 7$?

   (c) What lower and upper bounds can you give for a particular solution that satisfies $y(0) = 9$?

2. Consider the autonomous differential equation $\frac{dy}{dt} = |y|$.

   (a) What are the equilibrium solutions?

   (b) For what values of y does a solution exist?

   (c) For what values of y is there a unique solution?

   (d) Find all solutions, and sketch the family of solutions. *Hint*: Consider the cases $y > 0$ and $y < 0$ separately, and separate variables. Then consider the case $y = 0$.

3. Consider the autonomous differential equation $\frac{dy}{dt} = \frac{1}{(1+y)^2}$.

   (a) For what values of y is there a unique solution?

   (b) Find all solutions. *Hint*: Are there any equilibrium solutions? Now separate variables!

   (c) Find the particular solution $y(t)$ such that $y(0) = 1$. What is the largest interval of t-values on which this solution exists? Sketch the solution.

4. Consider the autonomous differential equation $\frac{dy}{dt} = 1 + y^2$.

   (a) For what values of y is there a unique solution?

   (b) Find all solutions.

   (c) Find the particular solution $y(t)$ such that $y(0) = 0$. What is the largest interval on which this solution exists? *Hint*: What is the domain of the solution?