# PARTIAL FRACTIONS:

example: $$\frac{1}{(y+1)(y-3)} = \frac{A(y-3)}{(y+1)(y-3)} + \frac{B\,(y+1)}{(y-3)(y+1)}$$

$$1 = A(y-3) + B(y+1) \quad \text{holds for all } y$$

If $y=3$: $1 = A\cancel{(3-3)}^{0} + B\,(3+1) \longrightarrow 1 = 4B$, so $B = \frac{1}{4}$

If $y=-1$: $1 = A\,(-1-3) + B\,\cancel{(-1+1)}^{0} \longrightarrow 1 = -4A$, so $A = \frac{-1}{4}$

$$\boxed{\frac{1}{(y+1)(y-3)} = \frac{\frac{-1}{4}}{y+1} + \frac{\frac{1}{4}}{y-3}}$$

# SLOPE FIELDS: qualitative analysis of solutions to $\frac{dy}{dt} = f(t,y)$

EXAMPLE: $\frac{dy}{dt} = -\frac{t}{y}$ so $f(t,y) = \frac{-t}{y}$

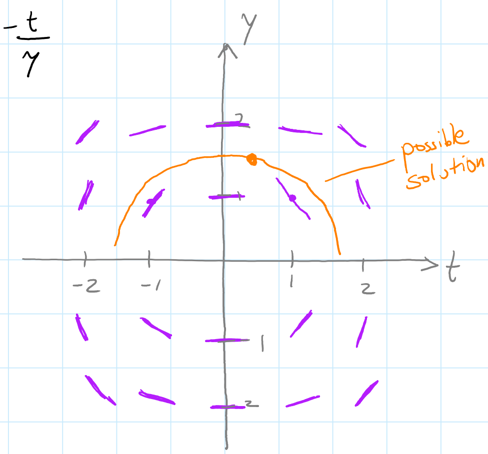
$y\frac{dy}{dt} = -t$

$f(1,1) = \frac{-1}{1} = -1$ $\qquad \left.\frac{dy}{dt}\right|_{(1,1)} = -1$

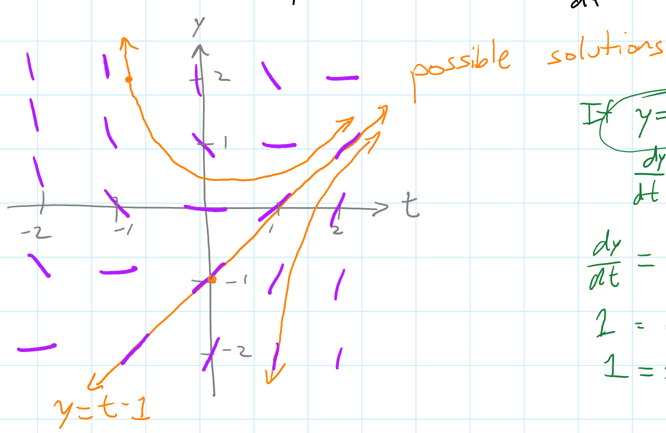
$f(0,1) = \frac{-0}{1} = 0$

$f(-1,1) = -\frac{-1}{1} = 1$

$f(1,0) = \frac{-1}{0}$ undefined




# PROBLEM: sketch the slope field for $\frac{dy}{dt} = t - y$




If $y = t-1$, then $\frac{dy}{dt} = 1$, so:

$$\frac{dy}{dt} = t - y$$

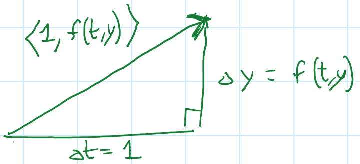
$$1 = t-(t-1)$$

$$1 = 1 \checkmark$$

## MATHEMATICA:

$f(t,y)$ window in ty-plane

```
VectorPlot[ {1, t - y}, {t, -5, 5}, {y, -5, 5},
 VectorStyle -> "Segment", VectorScale -> {Tiny, Tiny, None}]
```




slope: $\dfrac{f(t,y)}{1}$

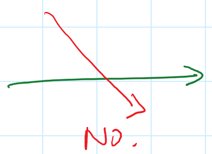
## DESMOS:

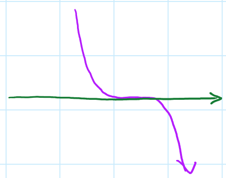
https://www.desmos.com/calculator/p7vd3cdmei

## GEOGEBRA:

https://www.geogebra.org/m/W7dAdgqc

## QUESTION: Can a solution cross an equilibrium solution?

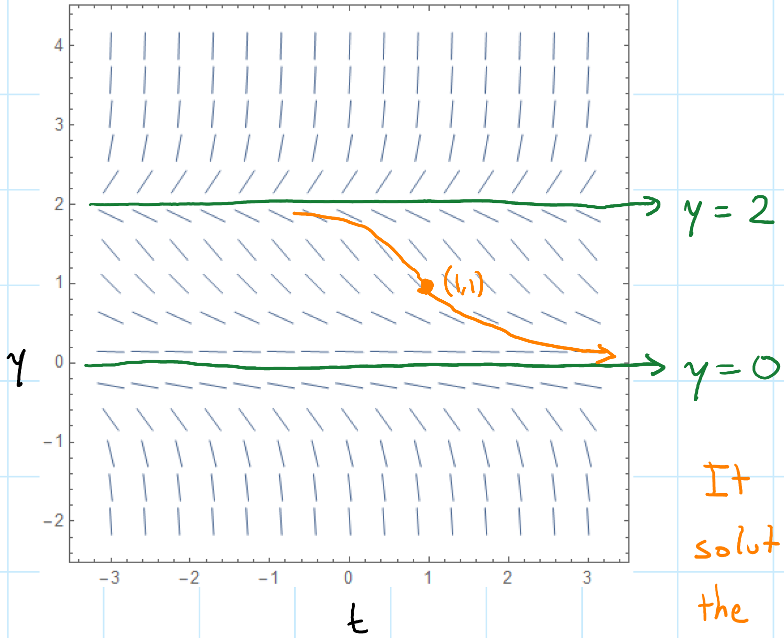



We will come back to this...

SOLUTIONS TO SLOPE FIELD PROBLEMS

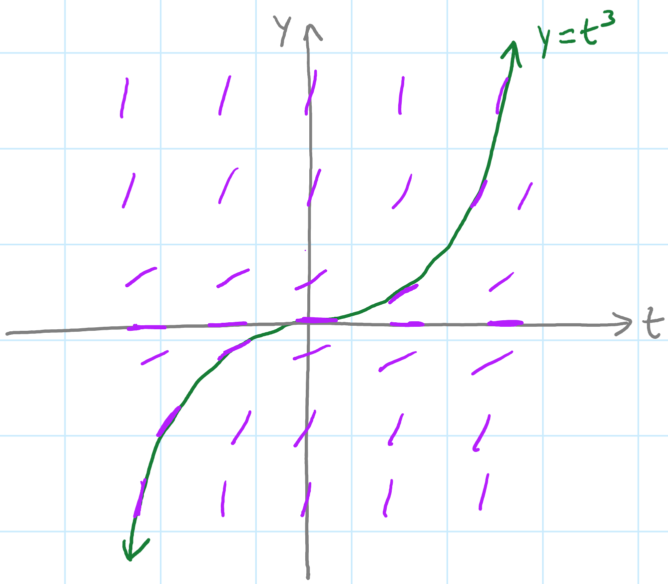
1. $\frac{dy}{dt} = y^3 - 2y^2$




equilibrium solutions

It appears that the solution through (1,1) approaches the equilibrium solution $y = 0$ as $t \to \infty$.

2. We know $y(t) = t^3$ is a solution to $\frac{dy}{dt} = f(y)$.

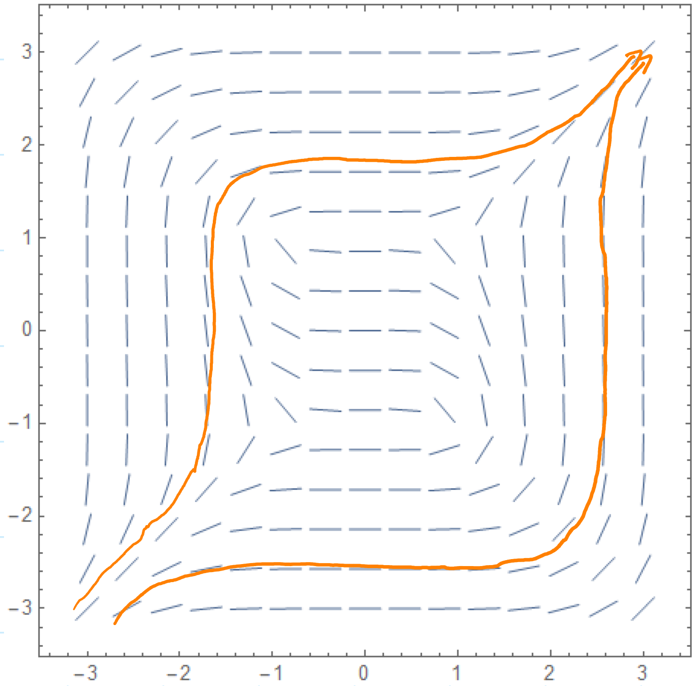



This equation is autonomous, meaning that the right side depends only on $y$, not on $t$.

Thus, the slopes for any y-value are the same!

3 $dy$ $t^4$

3. $\frac{dy}{dt} = \frac{t^4}{y^4 - 1}$




It looks like solutions approach the line $y = t$ as $t \to \infty$.