Separation of Variables

Math 230

1. Consider the differential equation $\frac{dy}{dt} = \frac{15 - y}{20}$.

(a) Find the general solution to the differential equation.

(b) Find the particular solution with initial condition $y(0) = 20$.

(c) Find the particular solution with initial condition $y(0) = 15$.

2. Find the general solution to $\frac{dy}{dt} = y^2 t$.

Have you possibly overlooked one (equilibrium) solution? Why or why not?

3. Find the general solution to $\dfrac{dy}{dt} = y(y+2)$.

4. Suppose that a new industry starts up river from a lake at $t = 0$ days, and this industry starts dumping a toxic pollutant into the river at a rate of 7 g/day. The flow of the river is 1000 m^3/day into the lake, which maintains a constant volume of $400,000$ m^3. The water in the lake exits through another river. We assume that all quantities are well-mixed and that there are no time delays for the pollutant reaching the lake from the river.

(a) Identify the quantities in the above scenario.

(b) Set up a differential equation to model the rate of change of change of the amount of pollutant in the lake. *Hint:*

$$\text{rate of change of amount of pollutant} = \text{rate flowing in} - \text{rate flowing out}$$

(c) Identify any equilibrium solutions of your differential equation

(d) Solve the differential equation to get a general solution.

(e) Using data from the problem, find the values of the constants that appear in the general solution. You then obtain the particular solution.

(f) What is the eventual amount of pollution in the lake? Convert this value to a concentration in mg/m^3. If the pollutant is toxic to fish, it is important to know the long-term steady-state concentration.