

MATH 220

12 May 2025

RECALL: basis is a linearly independent spanning set
dimension is the number of elements in a basis

Standard basis for $\mathbb{R}^4$: $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

$$\dim(\mathbb{R}^4) = 4$$

set of vectors

Standard basis for $\mathbb{R}^{2 \times 2}$: $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

$$\dim(\mathbb{R}^{2 \times 2}) = 4$$

Standard basis for $\mathbb{P}^3$: $\{1, x, x^2, x^3\}$

$$\dim(\mathbb{P}^3) = 4$$

polynomials, degree ≤ 3

$$a + bx + cx^2 + dx^3$$

Standard basis for $\mathbb{P}$: $\{1, x, x^2, x^3, x^4, x^5, x^6, \dots\}$

$$\dim(\mathbb{P}) = \infty$$

polynomials of any degree

$$a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$$

infinitely many terms!

$\mathbb{P}$ is an infinite-dimensional vector space!