Exam 3 Guidance

MATH 220 E • Spring 2025

The following is intended to help you focus your studying for Exam 3. Overall, you should know the material we have covered through Monday, April 28. While this exam will emphasize material since the last exam, this builds on what we studied in the first few weeks of the course, so it's clearly necessary to know all the material well.

More specifically, the following list gives things you should know. I don't claim for this list to be exhaustive, but it should help you organize your studying. If you have questions about this list or about any topics, *now* is the time to formulate and ask those questions! I want to help.

You should:

- You should understand what the row space, column space, and null space of a matrix are and be able to explain why they are all subspaces. You should be able to find these (especially when it comes to basis and dimension, see below).
- You should be able to decide if a given vector *is in* the row space, the column space, or the null space of a matrix with explanation.
- You should know the definition of both rank and nullity. You should know the Rank-Nullity Theorem and be able to use it to say things about the rank and nullity of certain given matrices.
- You should understand what the determinant of a matrix is and how it is defined (pick your favorite row/column).
- You should understand properties of determinants, such as Theorem 5.12. You should be aware of some tempting "properties" that don't work.
- You should understand what row operations do to the determinant of a matrix. This is the point of Mathematica Lab #8.
- You should understand what the determinant says about a matrix being invertible or not.
- You should understand and be able to use the definition of eigenvector and eigenvalue. Note: an eigenvalue may be 0 but an eigenvector may not be $\mathbf{0}$.
- You should be able to find the characteristic polynomial of a matrix and use it to find the eigenvalues.
- You should be able to find the eigenvectors of a matrix. You should know *why* the process works the way it does, not just *how* to do the process. You should know what an eigenspace is and how to find a basis for an eigenspace.
- You should know what the algebraic multiplicity and geometric multiplicity of an eigenvector and how to find these quantities.
- You should understand the connections between the various parts of the Unifying Theorem and be able to explain these connections.
- You should know what it means for a matrix to be diagonalizable and why diagonalization is

useful (powers of matrices).

- You should know how to determine whether or not a matrix is diagonalizable. If a matrix A is diagonalizable, you should be able to find matrices P and D such that $A = PDP^{-1}$.
- You should know what the dot product is, algebraic rules for it, and how it is used. You should know what length/norm means for a vector and how to tell if two vectors are orthogonal.
- You should understand what the orthogonal complement of a subspace S is and how to *check* if a vector is in $S^{\perp}$.
- Given a subspace S, you should be able to find a basis for $S^{\perp}$.
- You should know what an orthogonal set/basis is and how to check if a given basis is orthogonal. You should know what an orthonormal set/basis is and how to check if a given basis is orthonormal. You should know why these are important.
- You should understand Theorems 8.10, 8.11, 8.13.
- You should know how to compute projections of a vector $\mathbf{u}$ onto a subspace S and what that really means. You should be able to write a vector as a sum of a vector and S and vector in $S^{\perp}$. *You must remember that you need an orthogonal basis for S in order to use the formula for the projection.*
- You should know that a projection $\text{proj}_S(\mathbf{u})$ is the vector in S that is "closest" to $\mathbf{u}$.
- You should understand what the least squares method is trying to do and how to apply it to a given matrix A. In particular, you should be able find the "best fit" line ~~/quadratic/equation~~ to small set of data ~~but~~ by finding a system and using the least squares method. we didn't do this

Problems

Here are a few problems that I think you should be readily able to solve and explain, without much hesitation. Use these however you want. They do not represent every kind of question you may be asked.

- **Section 4.3** #1, 3, 7, 11, 13, 15
- **Section 5.1** #11, 13, 31, 35, 37
- **Section 5.2** #3, 7, 13, 23
- **Section 6.1** #23, 27, 33, 35, 47
- **Section 6.2** #13, 15, 17, 21, 25
- **Section 8.1** #15, 17, 31, 35, 43
- **Section 8.2** #3, 5
- **Section 8.5** #3, 5, 9
- **All sections:** As many of the TRUE/FALSE exercises as you can do. Treat them all as *always*, *sometimes*, or *never* true questions.
- **All sections:** As many of the "find an example" exercises as you can do.

What can I use on the exam?

- You may use one 4×6-inch card of notes (one side) prepared in advance.
- The exam will involve only minimal row-reduction and no tedious arithmetic. Calculators are *not necessary*. However, you may use a calculator, but only for row-reducing matrices or simple arithmetic, and you must state where exactly you used your calculator. You may not use a phone or computer.

How should I study?

First, understand that people learn differently and process information in different ways and at different speeds. I suggest:

- Read through each section again and think about whether or not the main theorems make intuitive sense. Can you explain them out loud?
- Do a few problems each day, ramping up as we get closer to the exam day. Talk with your classmates about the problems. Talk with me and visit the help sessions if you want to make sure things are correct or if you want to chat.
- Work on fluency! The exam is timed and you want to be able to do some of the problems efficiently. Perhaps give each other a few selected problems from a few different sections and time yourselves. This way, you won't know which section the problem came from.
- COME VISIT ME AND ASK QUESTIONS.