Linear Algebra – Day 33

MATH 220

1. If $T(\mathbf{x}) = A\mathbf{x}$ is one-to-one, *why* can't $\lambda = 0$ be an eigenvalue of A?

☞ Explain without reference to the Unifying Theorem.

2. If $\lambda = 0$ is an eigenvalue of matrix A, *why* is A not invertible?

3. If A is a diagonalizable matrix, how does $\det(A)$ relate to the eigenvalues of A?

☞ In fact, the relation also holds for non-diagonalizable square matrices, but this is harder to see.

4. If S is a subspace and $\mathbf{u} \in S$, then what is $\operatorname{proj}_S \mathbf{u}$?

5. If matrix A is diagonalizable, is there more than one matrix P such that $P^{-1}AP$ is a diagonal matrix? How many different matrices P are possible?

6. Find all possible ranks of matrices of the form

$$\begin{bmatrix} 0 & 0 & 0 & 0 & a_{15} \\ 0 & 0 & 0 & 0 & a_{25} \\ 0 & 0 & 0 & 0 & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \end{bmatrix}.$$

7. Let X_n be the $n \times n$ matrix that has 0s everywhere except on its two diagonals, where it has 1s. For example, here is X_5:

$$X_5 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Express $\text{col}(X_n)$ and $\text{null}(X_n)$ as spans.

8. Can a matrix be diagonalizable but not invertible? Can a matrix be invertible but not diagonalizable? Explain or give examples.

9. For a matrix A, show that $(\text{col}(A))^{\perp} = \text{null}(A^T)$.