Linear Algebra – Day 32

MATH 220

1. Suppose $S = \text{span}\left(\begin{bmatrix}1\\2\\1\end{bmatrix}, \begin{bmatrix}1\\0\\-1\end{bmatrix}\right)$.

(a) The two vectors you see above form a basis for S. Is this basis an orthogonal basis?

(b) Write the vector $\mathbf{u} = \begin{bmatrix}4\\3\\1\end{bmatrix}$ as the sum of a vector in S and a vector in $S^{\perp}$.

2. Suppose $S = \text{span}\left(\begin{bmatrix}2\\-1\\3\end{bmatrix}, \begin{bmatrix}2\\1\\-1\end{bmatrix}\right)$.

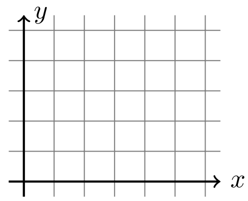
(a) The two vectors you see above form a basis for S. Is this basis an orthogonal basis?

(b) Write the vector $\mathbf{u} = \begin{bmatrix}2\\5\\-1\end{bmatrix}$ as the sum of a vector in S and a vector in $S^{\perp}$.

3. Milo: Hey Marissa! I collected some data for my big research project. So far, I have three points: $\{(1,1),(3,4),(6,5)\}$.

Marissa: A good start! It's a shame your three points are not on a common line.

Quick task: Plot the points and visually confirm they are not on the same line.



Milo: Wow, that's too bad. I was *really* hoping that a line would fit my data.

Marissa: You give up too quickly, Milo. Just pretend, for a moment, that the three data points *actually do* fit on a single line with equation $y = mx + b$.

Milo: Alright, I'll go along with your idea even though it clearly won't work.

Group chat: If Milo pretends the three points were all on the line $y = mx + b$, then what three equations need to be satisfied by b and m?

☞ One of the equations is $6m + b = 5$.

Marissa: See? You can now write your three equations as $A\mathbf{x} = \mathbf{y}$.

Group chat: What are A, $\mathbf{x}$, and $\mathbf{y}$?

Group chat: Is $A\mathbf{x} = \mathbf{y}$ a consistent system or an inconsistent system? Use row reduction to confirm this.

Milo: See? We can't do anything! The system has no solutions. We will never find a vector $\mathbf{x}$ where $A\mathbf{x}$ will equal $\mathbf{y}$.

Marissa: Well, that's true. We can never actually get $\mathbf{y}$. So, let's just try to find a vector to "plug in" for $\mathbf{x}$ that gets us *as close to* $\mathbf{y}$ *as possible.*

Group chat: Discuss Marissa's idea. If we can do what Marissa suggests, what are we really finding?

STOP *and wait for further instructions*

4. Since there is no $\mathbf{x}$ for which $A\mathbf{x} = \mathbf{y}$, we seek a vector $\hat{\mathbf{x}}$ so that $A\hat{\mathbf{x}}$ is *closest* to $\mathbf{y}$.

(a) Compute $\hat{\mathbf{x}} = (A^T A)^{-1} A^T \mathbf{y}$.

☞ Use Mathematica

(b) What is the equation of the line that best fits the data points?

(c) Sketch the line as best as you can on your plot above.