Linear Algebra – Day 29

MATH 220

For this entire page: $M = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 4 \end{bmatrix}$ and $N = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$.

1. Review: What are the eigenvalues of M and of N?

Answer: M is triangular, so the eigenvalues are on the diagonal: $\lambda = 3, 3, 4$

N is also triangular, so the eigenvalues are on the diagonal: $\lambda = 3, 3, 4$

For each matrix, we have $\text{alg}(3) = 2$ and $\text{alg}(4) = 1$.

2. For matrix M: Recall that last time, we found $\text{geo}(3) = 2$ and $\text{geo}(4) = 1$.

Someone at the table: Use Mathematica to compute `Eigensystem[ M ]` to find a basis for E_3 and a basis for E_4.

You will have to input the matrix M first.

3. For matrix N: Last time, we found $\text{geo}(3) = 1$ and $\text{geo}(4) = 1$.

Use Mathematica to find a basis for E_3 and a basis for E_4.

Or do it by hand if you've gotten fast with it!

and wait for further instructions

4. Erez: Oh wow! So we can conclude from our work above that M is diagonalizable and that N is *not* diagonalizable. That's a shame about N.

Group chat: Why? Also, find a P and D so that $M = PDP^{-1}$:

$$P = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \qquad \text{and} \qquad D = \begin{bmatrix} & 0 & 0 \\ 0 & & 0 \\ 0 & 0 & \end{bmatrix}$$

5. Ava: Simon, look at this matrix!

$$C = \begin{bmatrix} 1 & 1 & 5 & 8 & 4 & 0 & 6 & 2 & 4 \\ 0 & 2 & \pi & 7 & 1 & 8 & 2 & 4 & 9 \\ 0 & 0 & 3 & 3 & 4 & 8 & 2 & 3 & 6 \\ 0 & 0 & 0 & 4 & \sqrt{5} & 8 & 2 & 8 & 9 \\ 0 & 0 & 0 & 0 & 5 & 8 & 2 & 2 & 7 \\ 0 & 0 & 0 & 0 & 0 & 6 & 2 & 4 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 7 & 2\pi & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Simon: Wow, C is 9×9, so it is going to take forever to find all the eigenspaces.

Ava: But we already know that C is definitely diagonalizable! Cool!

Group chat: How did Ava know that C is diagonalizable without having to do a single computation?

Group chat: If $C = PDP^{-1}$ where D is diagonal, what could the matrix D equal?

6. Suppose A is a (random) 6×6 matrix whose eigenvalues are $\lambda = 2, 2, 3, 3, 3, 4$. With you group, come up with a strategy that will allow you to determine whether or not A is diagonalizable as quickly as possible.

7. Let $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$. Each task below requires you to solve the problem posed above it.

(a) What is the characteristic polynomial of A?

(b) What are the eigenvalues of A?

(c) For each eigenvalue λ, find a basis for E_λ.

(d) Determine if A is diagonalizable or not (give a reason).

(e) Find a P and diagonal D for which $A = PDP^{-1}$.

(f) Find a formula for A^m.

8. In part (b) of previous problem, a former student made an algebra mistake and ended up saying that $\lambda = 1$ was an eigenvalue of A even though $\lambda = 1$ is actually not an eigenvalue of A. You might think that this poor, unfortunate student is now doomed to get all the rest of the problems wrong.

Group chat: What will happen in part (c) that should alert the student that they did something wrong?